Problem Set 1

Due: 10:00 a.m. on Thursday, September 12

Instructions: Submit solutions to all of the following exercises. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. You may submit your solutions either in-class or to the Department of Mathematics (*with date and time of submission noted*).

Exercises: Be sure to show all of your work and fully justify your answers and reasoning.

1. Verify the identity $A \cup (A \cap B) = A$ via
   (a) a Venn Diagram;
   (b) formal proof via First Principles (i.e., show that two sets are equal).
2. Prove De Morgan's Law $\overline{A \cap B} = \overline{A} \cup \overline{B}$ from First Principles.
3. Show that $R = \{(x, y) \in \mathbb{Z} \times \mathbb{Z} \mid x + y = 1\}$ is not a transitive relation on $\mathbb{Z}$. Is it symmetric? Reflexive?
4. Let S be the set of all polynomials in the variable x with real coefficients. Define the relation
$$R = \{(f, g) \in S \times S \mid f' = g'\}$$
   where f' is the derivative of f.
   (a) Show that R is an equivalence relation on S.
   (b) Determine the equivalence class of $p(x) = x^2 + 3x$.
5. Define the function $f : \mathbb{R} \to \mathbb{R}$ by $f(x) = \dfrac{x-2}{x+1}$.
   (a) Using the definition of one-to-one, show that f is one-to-one (i.e., show that $f(x_1) = f(x_2) \implies x_1 = x_2$).
   (b) Determine the inverse function from the range of f to the domain of f.
   (c) State the domain and range of f.
   (d) Determine whether f is onto $\mathbb{R}$.