MATH 2090, Fall 2018 **Dr. S. Cooper**

Tutorial Worksheet #6

Monday, October 29

Name and Student Number: ______________________________

Work the following exercises. *Explain all of your reasoning.* Remember to use good notation and full sentences. All cited exercises are from the textbook *Linear Algebra* by Larry Smith.

1. (Section 11.4 #4) Let $T : \mathbb{R}^4 \to \mathbb{R}^7$ be the linear transformation whose matrix relative to the standard bases is

$$A = \begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 4 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & -1 & 3 & 1 \\ 3 & 2 & -5 & 0 \\ 0 & 0 & 7 & 1 \\ 4 & 0 & 1 & 0 \end{bmatrix}$$

and let $S : \mathbb{R}^7 \to \mathbb{R}^3$ be the linear transformation whose matrix relative to the standard bases is

$$B = \begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find the matrix of $S \circ T : \mathbb{R}^4 \to \mathbb{R}^3$ relative to the standard bases. What is $(S \circ T)(1, -1, 1, -1)$?

2. (Section 11.4 #5) Let $S, T : \mathbb{R}^3 \to \mathbb{R}^2$ be the linear transformations with matrices

$$A = \begin{bmatrix} 6 & -1 & 2 \\ 2 & 4 & 1 \end{bmatrix}$$

and

$$B = \begin{bmatrix} -5 & 0 & -7 \\ 2 & -1 & 9 \end{bmatrix},$$

respectively, with respect to the standard bases. What is the matrix of the linear transformation

$$3S - 7T : \mathbb{R}^3 \to \mathbb{R}^2$$

with respect to the standard bases? Find $(3S - 7T)(1, 2, 3)$.

3. Let $\mathcal{B} = \left\{1, x, \frac{3}{2}x^2 - \frac{1}{2}, \frac{5}{2}x^3 - \frac{3}{2}x\right\}$, and let $\mathcal{C} = \{1, x, x^2, x^3\}$ be bases for $\mathcal{P}_3(\mathbb{C})$. (The vectors in the basis $\mathcal{B}$ are the first four *Legendre polynomials*. These arise in physics when solving Laplace's equation in spherical coordinates.) Find the coordinate vectors with respect to $\mathcal{B}$ of the following polynomials. [*Hint: It might save you some time to first find the change of coordinate matrix* $P_{\mathcal{B}\leftarrow\mathcal{C}}$, *but you don't have to do so to answer the question.*]

   (a) $1 + x + x^2 + x^3$.

   (b) $2 - ix + (1+i)x^3$.

   (c) $\frac{3}{2} + \frac{5i}{2}x - \frac{3}{2}x^2 - \frac{5i}{2}x^3$.

4. Consider the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{2(x+y+z)}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

   Geometrically, this is a reflection in $\mathbb{R}^3$ about the plane $x + y + z = 0$. Let $\mathcal{B}$ be the standard basis for $\mathbb{R}^3$. Recall that on Worksheet #5, you showed that

$$[T]_{\mathcal{B}}^{\mathcal{B}} = \begin{bmatrix} 1/3 & -2/3 & -2/3 \\ -2/3 & 1/3 & -2/3 \\ -2/3 & -2/3 & 1/3 \end{bmatrix}.$$

   (a) Let $\mathcal{C} = \left\{\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 0 \\ 1 \end{pmatrix}, \begin{pmatrix} -1 \\ 1 \\ 0 \end{pmatrix}\right\}$ be another basis for $\mathbb{R}^3$. Find the change-of-coordinates matrices $P_{\mathcal{C}\leftarrow\mathcal{B}}$ and $P_{\mathcal{B}\leftarrow\mathcal{C}}$.

   (b) Find the matrix $[T]_{\mathcal{C}}^{\mathcal{C}}$. [*Hint: You don't have to use part (b), but it might save you some time.*]

5. (Section 11.4 #11) Let $T : \mathbb{R}^3 \to \mathbb{R}^3$ be the linear transformation whose matrix relative to the standard basis is

$$A = \begin{bmatrix} 1 & 1 & 1 \\ -1 & 1 & 0 \\ 1 & 1 & 0 \end{bmatrix}.$$

   Let $\mathcal{C} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ be the basis of $\mathbb{R}^3$ where

$$\mathbf{v_1} = (1, 1, 1)$$
$$\mathbf{v_2} = (1, 1, 0)$$
$$\mathbf{v_3} = (1, 0, 0).$$

   Find $[T]_{\mathcal{C}}^{\mathcal{C}}$.