MATH 2090, Fall 2018 **Dr. S. Cooper**

Tutorial Worksheet #5

Monday, October 22

Name and Student Number: ________________________________

Work the following exercises. *Explain all of your reasoning.* Remember to use good notation and full sentences. All cited exercises are from the textbook *Linear Algebra* by Larry Smith.

1. Let A be an $m \times n$ matrix with entries from the field $\mathbb{F}$. Prove that $T : \mathbb{F}^n \to \mathbb{F}^m$ defined by

$$T(\mathbf{x}) = A\mathbf{x}$$

for all $\mathbf{x} \in \mathbb{F}^n$ is a linear transformation.

2. Consider the linear mapping $T : \mathbb{R}^3 \to \mathbb{R}^3$ given by

$$T\left(\begin{pmatrix} x \\ y \\ z \end{pmatrix}\right) = \begin{pmatrix} x \\ y \\ z \end{pmatrix} - \frac{2(x+y+z)}{3}\begin{pmatrix} 1 \\ 1 \\ 1 \end{pmatrix}.$$

Geometrically, this is a reflection in $\mathbb{R}^3$ about the plane $x + y + z = 0$. Determine the matrix of T with respect to the standard basis $\mathcal{B}$ of $\mathbb{R}^3$. That is, find $[T]_{\mathcal{B}}^{\mathcal{B}}$.

3. (Section 11.4 #2(4)) Find the matrix of the linear transformation $T : \mathcal{P}_2(\mathbb{R}) \to \mathcal{P}_3(\mathbb{R})$ defined by

$$T(p(x)) = xp(x)$$

relative to the standard bases of $\mathcal{P}_2(\mathbb{R})$ and $\mathcal{P}_3(\mathbb{R})$.

4. (Section 11.4 #3) Let $T : \mathbb{R}^4 \to \mathbb{R}^7$ be the linear transformation whose matrix relative to the standard bases is

$$\begin{bmatrix} 1 & 2 & -1 & 0 \\ 0 & -1 & 4 & 1 \\ 2 & 0 & 2 & 0 \\ 0 & -1 & 3 & 1 \\ 3 & 2 & -5 & 0 \\ 0 & 0 & 7 & 1 \\ 4 & 0 & 1 & 0 \end{bmatrix}.$$

Find $T((1, 2, 3, 4))$.

5. (Section 11.4 #9) Consider the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by

$$T((x, y, z)) = (3y + 4z, 3x, 4x).$$

(a) Find the matrix of T with respect to the standard bases of $\mathbb{R}^3$.

(b) Find the matrix of T with respect to the basis $\mathcal{B} = \{(0,4,-3),(5,3,4),(5,-3,-4)\}$ of $\mathbb{R}^3$. That is, find $[T]_{\mathcal{B}}^{\mathcal{B}}$.

6. (Section 11.4 #8 modified) Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation

$$T((x,y)) = (x+3y, 3x+y).$$

The set $\mathcal{B} = \{(1,1),(1,-1)\}$ is a basis for $\mathbb{R}^2$.

(a) Find the matrix $[T]_{\mathcal{B}}^{\mathcal{B}}$.

(b) Let $\mathcal{C} = \{(1,0),(0,1)\}$ be the standard basis of $\mathbb{R}^2$. Find the matrix $[T]_{\mathcal{B}}^{\mathcal{C}}$.

7. Let $T : V \to W$ be a linear transformation and $\mathcal{B} = \{\mathbf{v_1}, \ldots, \mathbf{v_n}\}$ and $\mathcal{C} = \{\mathbf{w_1}, \ldots, \mathbf{w_m}\}$ be bases for V and W, respectively. Let $A = [T]_{\mathcal{B}}^{\mathcal{C}}$. Prove that $\mathbf{v} \in \ker(T)$ if and only if $[\mathbf{v}]_{\mathcal{B}} \in \mathrm{Null}(A)$.

8. Let $T : M_{2\times 2}(\mathbb{R}) \to \mathcal{P}_2(\mathbb{R})$ be the linear transformation defined by

$$T\left(\begin{bmatrix} a & b \\ c & d \end{bmatrix}\right) = (a-3b+d) + (-2a+6b+2c+2d)x + (2a-6b+2d)x^2.$$

(a) Find the matrix of T with respect to the standard bases for $M_{2\times 2}(\mathbb{R})$ and $\mathcal{P}_2(\mathbb{R})$.

(b) Find a basis for $\ker(T)$ and a basis for $\mathrm{Im}(T)$.