MATH 2090, Fall 2018 **Dr. S. Cooper**

Tutorial Worksheet #3

Monday, September 24

Name and Student Number: ______________________________

Work the following exercises. *Explain all of your reasoning.* Remember to use good notation and full sentences. All cited exercises are from the textbook *Linear Algebra* by Larry Smith.

1. Show that the following subsets are linearly independent.

   (a) $\{1 + x + x^2, 5 - 2x + 2x^2, -2 + 3x + x^2\} \subseteq \mathcal{P}_2(\mathbb{R})$

   (b) $\left\{ \begin{bmatrix} 1 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 2 & 0 \\ 0 & 0 & 0 \end{bmatrix} \right\} \subseteq M_{2\times 3}(\mathbb{R})$

   (c) $\left\{ \begin{bmatrix} i \\ 2i \end{bmatrix}, \begin{bmatrix} 2 \\ 1 \end{bmatrix} \right\} \subseteq \mathbb{C}^2$

2. Show that the following subsets are linearly dependent.

   (a) $\left\{ \begin{bmatrix} 1 \\ 1 \end{bmatrix}, \begin{bmatrix} 4 \\ 4 \end{bmatrix} \right\} \subseteq \mathbb{R}^2$

   (b) $\{1 + 3x + x^2, -1 + x^2, 5 + x, 3x^2\} \subseteq \mathcal{P}_2(\mathbb{R})$

   (c) $\left\{ \begin{bmatrix} 1 & 3 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 5 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 0 & 3 \end{bmatrix} \right\} \subseteq M_{2\times 2}(\mathbb{R})$

   (d) $\left\{ \begin{bmatrix} 1+i \\ i \end{bmatrix}, \begin{bmatrix} 2i \\ -1+i \end{bmatrix} \right\} \subseteq \mathbb{C}^2$

   (e) Any finite subset of a vector space that has two identical vectors.

3. Let $V = \{f : \mathbb{R} \to \mathbb{R} \mid F \text{ is continuous }\}$. You may accept without verification that V is a vector space over $\mathbb{R}$ with the following rules:

   $$(f+g)(x) = f(x) + g(x) \quad \text{and} \quad (\alpha f)(x) = \alpha f(x)$$

   for all $f, g \in V$ and $\alpha \in \mathbb{R}$. Prove that $\sin x$ and $\cos x$ are linearly independent in V. [*Hint:* Suppose that $a \sin x + b \cos x = 0$ and try plugging in different values for x.]

4. (Section 5.3 #5) Let $E = \{(1,1,0,0,1), (1,1,0,1,1), (0,1,1,1,1), (2,1,-1,0,1)\} \subseteq \mathbb{R}^5$ and $\mathcal{U} = Span(E)$. Find a linearly independent subset F of E such that $Span(F) = \mathcal{U}$.

5. The set

   $$\mathcal{U} = \left\{ \begin{bmatrix} a & b \\ c & d \end{bmatrix} : a + b = d \right\}$$

   is a subspace of $M_{2\times 2}(\mathbb{R})$.

(a) Find a basis for $\mathcal{U}$ (and prove that it is indeed a basis!).

(b) What is the dimension of $\mathcal{U}$?

6. (Section 5.3 #9) Suppose that E and F are finite sets of vectors in a vector space V with $E \subseteq F$. Prove that if E is linearly dependent, then so is F.

7. (Section 5.3 #10) Suppose that E and F are finite sets of vectors in a vector space V with $E \subseteq F$. Prove that if F is linearly independent, then so is E.

8. (Section 5.3 #17) Let E', E'' be linearly independent finite sets of vectors in a vector space V. Show that $E' \cap E''$ is linearly independent.

9. What is the dimension of $\mathcal{U} = \{p(x) \in \mathcal{P}_n(\mathbb{R}) : p(0) = 0\}$, as a subspace of $\mathcal{P}_n(\mathbb{R})$?