MATH 2090, Fall 2018 **Dr. S. Cooper**

Tutorial Worksheet #1

Monday, September 10

Name and Student Number: ____

Work the following exercises. *Explain all of your reasoning.* Remember to use good notation and full sentences. All cited exercises are from the textbook *Linear Algebra* by Larry Smith.

1. Show that $M_{2\times 3}(\mathbb{R})$ is a vector space over $\mathbb{R}$.

2. (Section 2.4 #9) Let V be the set of all ordered pairs of real numbers (a, b). Define an addition for the elements of V by the rule

$$(a, b) \oplus (c, d) = (a + c, b + d),$$

and a multiplication of elements of V by real numbers with the rule

$$r \cdot (c, d) = (rc, 0).$$

Is V with these two operations a vector space? Justify your answer.

3. (Section 2.4 #10) Let V and W be vector spaces over the field $\mathbb{F}$. Denote by $V \oplus W$ the set of all ordered pairs $(\mathbf{A}, \mathbf{B})$, where $\mathbf{A} \in V$ and $\mathbf{B} \in W$. Define an addition for the elements of $V \oplus W$ by the rule

$$(\mathbf{A}, \mathbf{B}) + (\mathbf{C}, \mathbf{D}) = (\mathbf{A} + \mathbf{C}, \mathbf{B} + \mathbf{D}),$$

and a multiplication of elements of $V \oplus W$ by scalars in $\mathbb{F}$ by the rule

$$r \cdot (\mathbf{A}, \mathbf{B}) = (r\mathbf{A}, r\mathbf{B}).$$

Is $V \oplus W$ with these two operations a vector space over $\mathbb{F}$? Justify your answer. We call $V \oplus W$ the *direct sum* of V and W.

4. Let $V = \{x \in \mathbb{R} \mid x > 0\}$ and define operations $\oplus$ and $\circ$ on V by $x \oplus y = xy$ and $t \circ x = x^t$ for all $t \in \mathbb{R}$.

   (a) Show that V is a vector space over $\mathbb{R}$ with these operations.

   (b) What is the additive inverse of the vector $\mathbf{x} = 4$?

5. Show that $V = \left\{ \begin{bmatrix} a & 0 \\ 0 & b \end{bmatrix} : a, b \in \mathbb{R} \right\}$ is a vector space over $\mathbb{R}$ under the usual operations of matrix addition and scalar multiplication.

6. Let $V = \{(a, b) \mid a, b \in \mathbb{R}, b > 0\}$ and define addition by $(a, b) \oplus (c, d) = (ad + bc, bd)$ and define scalar multiplication by $t \circ (a, b) = (tab^{t-1}, b^t)$. Prove that with these operations, V is a vector space over $\mathbb{R}$.

7. A *Fibonacci-type sequence* is a sequence of real numbers of the form $\{x_0, x_1, x_2, \ldots\}$ such that $x_n = x_{n-1} + x_{n-2}$ for $n \geq 2$. One famous Fibonacci sequence is $\{1, 1, 2, 3, 5, 8, 13, 21, \ldots\}$. Let V be the set of all Fibonacci-type sequences. Define addition on V by

$$\{x_0, x_1, x_2, \ldots\} + \{y_0, y_1, y_2, \ldots\} = \{x_0 + y_0, x_1 + y_1, x_2 + y_2, \ldots\}$$

and define scalar multiplication on V by real numbers with the rule

$$r \cdot \{x_0, x_1, x_2, \ldots\} = \{rx_0, rx_1, rx_2, \ldots\}.$$

Show that with these operations, V is a vector space over $\mathbb{R}$.