Dictionary Quiz 5 (B02 & B03)
Sample Solutions

Name and Student Number: _____

In the space provided, please write your solutions to the following exercises. *Fully explain your work.* Remember to use good notation and full sentences. For full credit you must also demonstrate serious effort on the Tutorial Worksheet.

Good Luck!

1. Let V be a vector space over the field $\mathbb{F}$.

   (a) Complete the following definition: [2 pts]

   An *inner product on V* is a function $\langle \ , \ \rangle : V \times V \to \mathbb{F}$ such that the following properties hold for all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in V and all α in $\mathbb{F}$:

      i. $\langle \mathbf{v}, \mathbf{w} \rangle = \overline{\langle \mathbf{w}, \mathbf{v} \rangle}$

      ii. **Solution:** $\langle \alpha\mathbf{v}, \mathbf{w} \rangle = \alpha \langle \mathbf{v}, \mathbf{w} \rangle$

     iii. $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle + \langle \mathbf{v}, \mathbf{w} \rangle$

     iv.   • $\langle \mathbf{v}, \mathbf{v} \rangle \geq 0$

               • **Solution:** $\langle \mathbf{v}, \mathbf{v} \rangle = 0$ if and only if $\mathbf{v} = 0$

   (b) Give an example of an inner product space. For full credit you must state the vector space V, the field $\mathbb{F}$, and define the inner product.   [2 pts]

   Solution: Let $V = \mathbb{C}^n, \mathbb{F} = \mathbb{C}$ and for $\mathbf{v} = (v_1, \ldots, v_n), \mathbf{w} = (w_1, \ldots, w_n) \in V$ define

   $$\langle \mathbf{v}, \mathbf{w} \rangle = v_1\overline{w_1} + \cdots + v_n\overline{w_n}.$$

   This is the standard Hermitian inner product.

2. You have demonstrated serious effort on the Tutorial Worksheet.   [1 pt]