

Dictionary Quiz 5 (B01)
Sample Solutions

Name and Student Number: _____

In the space provided, please write your solutions to the following exercises. *Fully explain your work.* Remember to use good notation and full sentences. For full credit you must also demonstrate serious effort on the Tutorial Worksheet.

Good Luck!

1. Let V be a vector space over the field \mathbb{F} .

(a) Complete the following definition: [2 pts]

An *inner product on V* is a function $\langle \cdot, \cdot \rangle : V \times V \rightarrow \mathbb{F}$ such that the following properties hold for all $\mathbf{u}, \mathbf{v}, \mathbf{w}$ in V and all α in \mathbb{F} :

i. **Solution:** $\langle \mathbf{v}, \mathbf{w} \rangle = \overline{\langle \mathbf{w}, \mathbf{v} \rangle}$

ii. $\langle \alpha \mathbf{v}, \mathbf{w} \rangle = \alpha \langle \mathbf{v}, \mathbf{w} \rangle$

iii. **Solution:** $\langle \mathbf{u} + \mathbf{v}, \mathbf{w} \rangle = \langle \mathbf{u}, \mathbf{w} \rangle + \langle \mathbf{v}, \mathbf{w} \rangle$

iv. • $\langle \mathbf{v}, \mathbf{v} \rangle \geq 0$

• $\langle \mathbf{v}, \mathbf{v} \rangle = 0$ if and only if $\mathbf{v} = \mathbf{0}$

(b) Give an example of an inner product space. For full credit you must state the vector space V , the field \mathbb{F} , and define the inner product. [2 pts]

Solution: Let $V = \mathbb{C}^n$, $\mathbb{F} = \mathbb{C}$ and for $\mathbf{v} = (v_1, \dots, v_n)$, $\mathbf{w} = (w_1, \dots, w_n) \in V$ define

$$\langle \mathbf{v}, \mathbf{w} \rangle = v_1 \overline{w_1} + \dots + v_n \overline{w_n}.$$

This is the standard Hermitian inner product.

2. You have demonstrated serious effort on the Tutorial Worksheet. [1 pt]