MATH 2090: Linear Algebra 2   **Dr. S. Cooper, Fall 2018**

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Dictionary Quiz 3 (B01)
Sample Solutions

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Name and Student Number: _____

In the space provided, please write your solutions to the following exercises. *Fully explain your work.* Remember to use good notation and full sentences. For full credit you must also demonstrate serious effort on the Tutorial Worksheet.

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Good Luck!

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1. Let A be an $m \times n$ matrix with entries in the field $\mathbb{F}$.

    (a) Complete the following definition: [2 pts]

    The *nullspace* (or *kernel*) of A, denoted $Null(A)$, is

    Solution:
    $$Null(A) = \{\mathbf{v} \in \mathbb{F}^n \mid A\mathbf{v} = \mathbf{0}_{\mathbb{F}^m}\}.$$

    (b) Give an example of a matrix A whose nullspace has dimension 2. For full credit, your answer must *briefly* justify that $\dim(Null(A)) = 2$. [Note: You do not need to find a basis for the nullspace to justify your answer.]   [2 pts]

    Solution: Let
    $$A = \begin{bmatrix} 1 & 2 & 0 & 0 & 1 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 & 3 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

    If we were to solve the system of equations $A\mathbf{v} = \mathbf{0}_{\mathbb{R}^4}$ then we would immediately see that we have 2 free variables. Since the number of free variables determines the dimension of the nullspace of A, we must have that $\dim(Null(A)) = 2$.

2. You have demonstrated serious effort on the Tutorial Worksheet.   [1 pt]