MATH 2090: Linear Algebra 2 **Dr. S. Cooper, Fall 2018**

Dictionary Quiz 2 (B02 & B03)
Sample Solutions

Name and Student Number: ____________________

In the space provided, please write your solutions to the following exercises. *Fully explain your work*. Remember to use good notation and full sentences. For full credit you must also demonstrate serious effort on the Tutorial Worksheet.

Good Luck!

1. Let V be a vector space over the field $\mathbb{F}$ (for us, $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$).

   (a) Complete the following definition: [2 pts]

   A set of vectors $\{\mathbf{v_1}, \ldots, \mathbf{v_k}\}$ in V is *linearly independent* if

   Solution: the only solution to the equation $a_1\mathbf{v_1} + \cdots + a_k\mathbf{v_k} = \mathbf{0}$ is

   $$a_1 = \cdots = a_k = 0 \in \mathbb{F}.$$

   (b) Give an example of a set of 3 vectors which is linearly independent. For full credit, your answer must explicitly state the vector space, the field $\mathbb{F}$ and explain why the example is linearly independent. [2 pts]

   Solution: Let $V = M_{2\times 2}(\mathbb{R})$ and $\mathbb{F} = \mathbb{R}$. The set

   $$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} \right\}$$

   is linearly independent. Indeed, if a, b and c are real scalars such that

   $$a \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + b \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + c \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}$$

   then

   $$\begin{bmatrix} a & b \\ c & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

   Thus, it must be the case that $a = b = c = 0$.

2. You have demonstrated serious effort on the Tutorial Worksheet. [1 pt]