MATH 2090: Linear Algebra 2   **Dr. S. Cooper, Fall 2018**

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Dictionary Quiz 2 (B01)
Sample Solutions

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Name and Student Number: _____

In the space provided, please write your solutions to the following exercises. *Fully explain your work.* Remember to use good notation and full sentences. For full credit you must also demonstrate serious effort on the Tutorial Worksheet.

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Good Luck!

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1. Let V be a vector space over the field $\mathbb{F}$ (for us, $\mathbb{F} = \mathbb{R}$ or $\mathbb{F} = \mathbb{C}$).

   (a) Complete the following definition: [2 pts]

   A set of vectors $\{\mathbf{v_1}, \ldots, \mathbf{v_k}\}$ in V is *linearly independent* if

   Solution: the only solution to the equation $a_1\mathbf{v_1} + \cdots + a_k\mathbf{v_k} = \mathbf{0}$ is

   $$a_1 = \cdots = a_k = 0 \in \mathbb{F}.$$

   (b) Give an example of a set of 3 vectors which is *not* linearly independent. For full credit, your answer must explicitly state the vector space, the field $\mathbb{F}$ and explain why the example is not linearly independent.   [2 pts]

   Solution: Let $V = M_{2\times2}(\mathbb{R})$ and $\mathbb{F} = \mathbb{R}$. The set

   $$\left\{ \begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} \right\}$$

   is not linearly independent. Indeed, we have the following equation where not all the scalars are 0:

   $$2\begin{bmatrix} 1 & 0 \\ 0 & 0 \end{bmatrix} + 1\begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix} + (-1)\begin{bmatrix} 2 & 1 \\ 0 & 0 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix}.$$

2. You have demonstrated serious effort on the Tutorial Worksheet.   [1 pt]