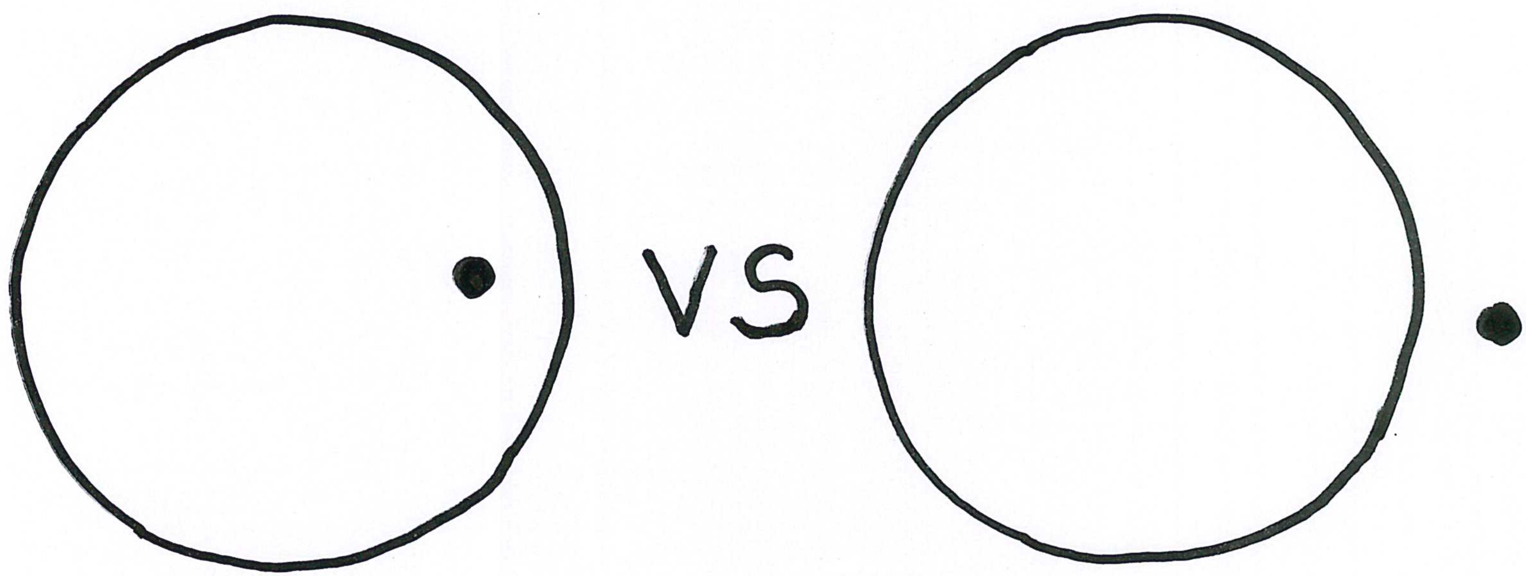
Recall:

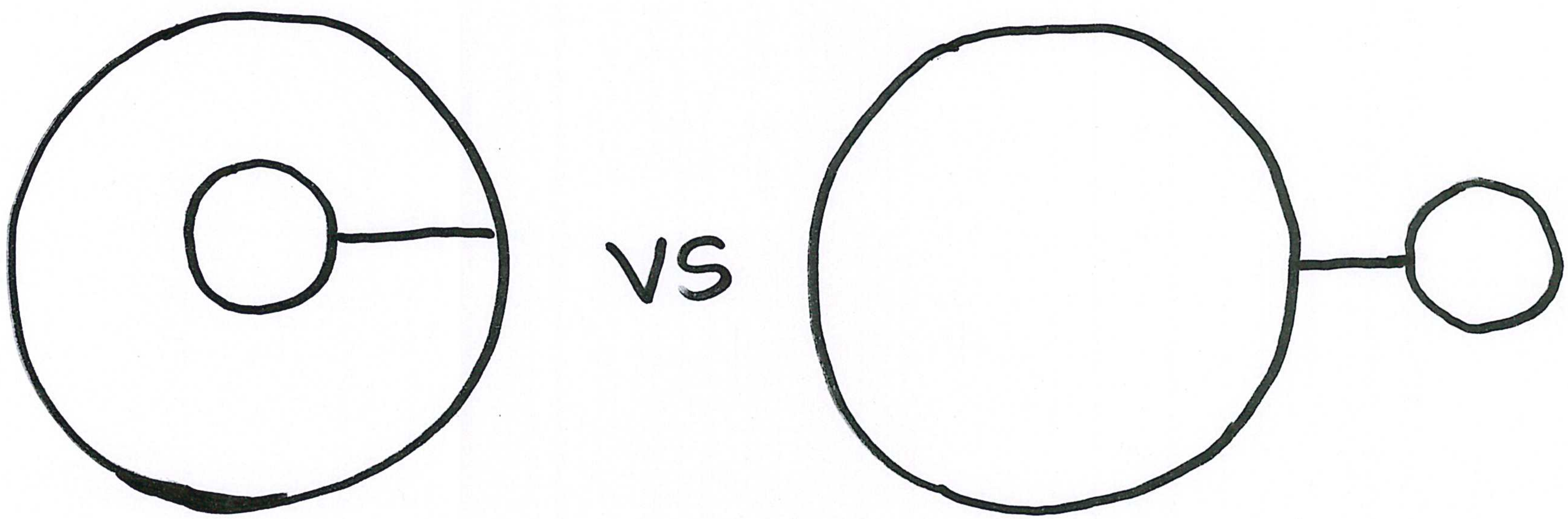
When are two objects considered "topologically the same"? ⤳ internal structure!

Two spaces are **homotopic** if we can continuously deform one of them into the other without cutting or pasting. (The deformation is called a homotopy.)

Example: Are these homotopic?



Example: Are these homotopic?

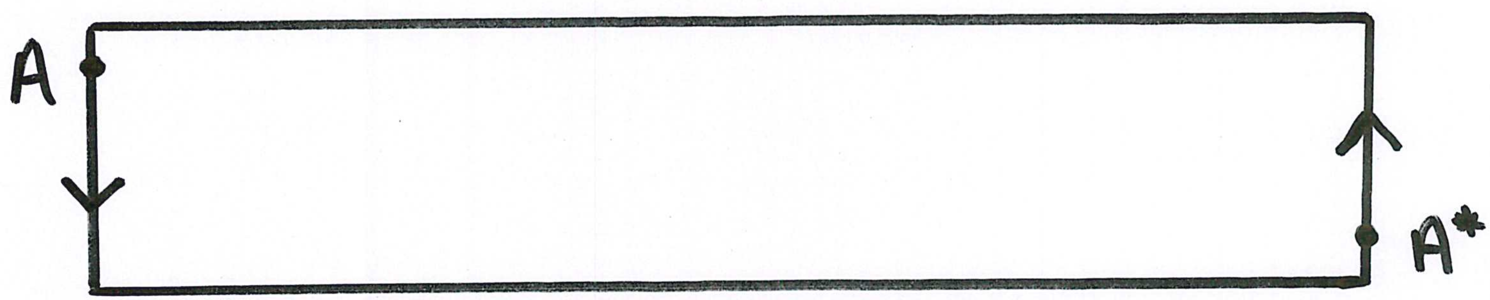


Two-Manifolds (Definitions, pages 230–231 & 234 of text)

- A **two-manifold** is a space that *feels locally* like the

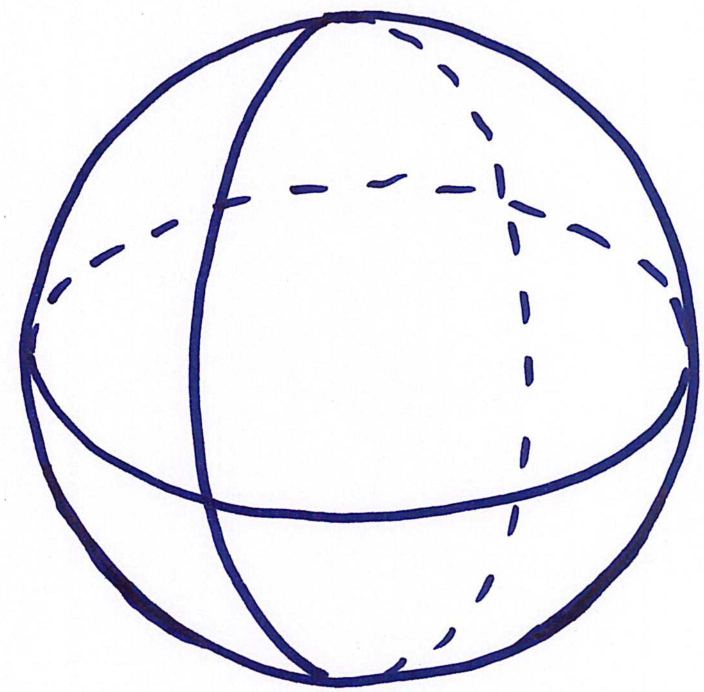
- The **genus** of a two-manifold is the maximal number of consecutive closed circular cuts we can make on that surface without

An Example Of A Non-Orientable Surface: Möbius Strip

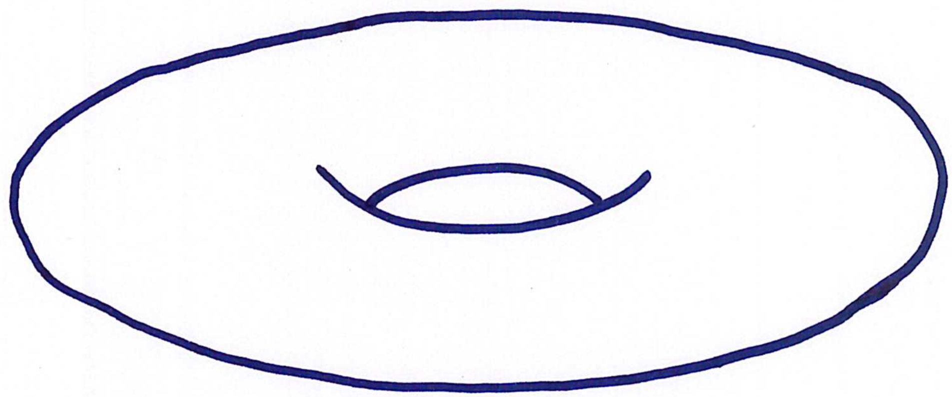


Note: The Möbius strip is

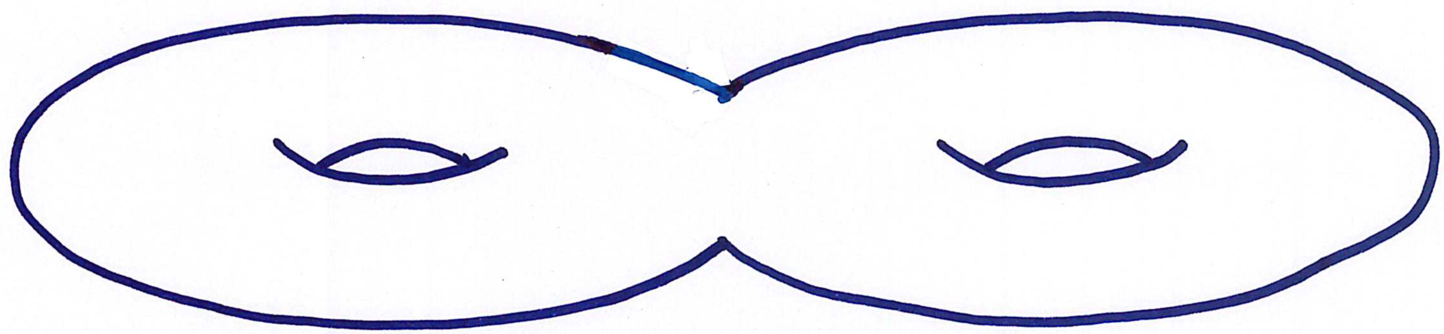
Orientable Two-Manifold Example: Sphere



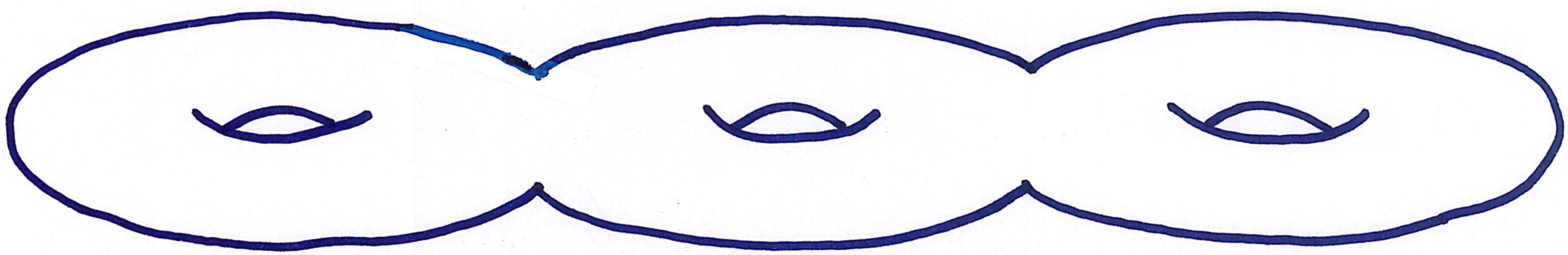
Orientable Two-Manifold Example: Torus



Orientable Two-Manifold Example: Connected Sum of Two Tori



Orientable Two-Manifold Example: Connected Sum of Three Tori



Euler Characteristic Of Two-Manifolds (page 234 of text)

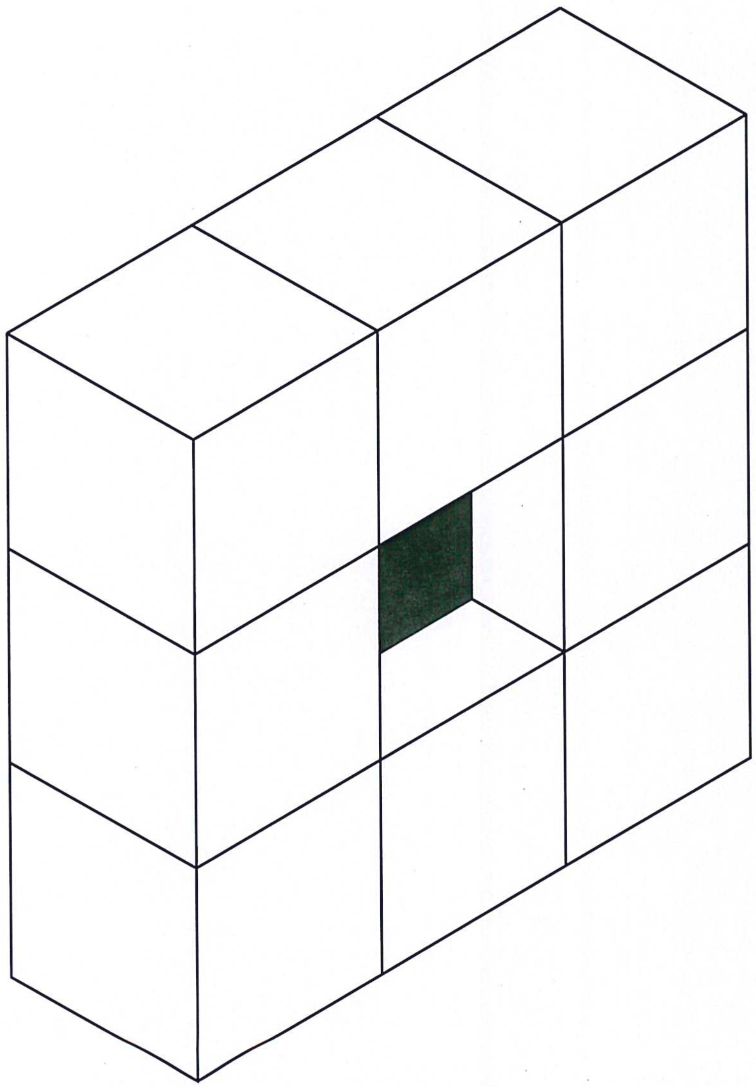
The **Euler characteristic** of a two-manifold is

$$V - E + F$$

where

- V is the number of vertices;
- E is the number of edges;
- F is the number of polygonal faces

Tiling a Torus, Euler characteristic



V =

E =

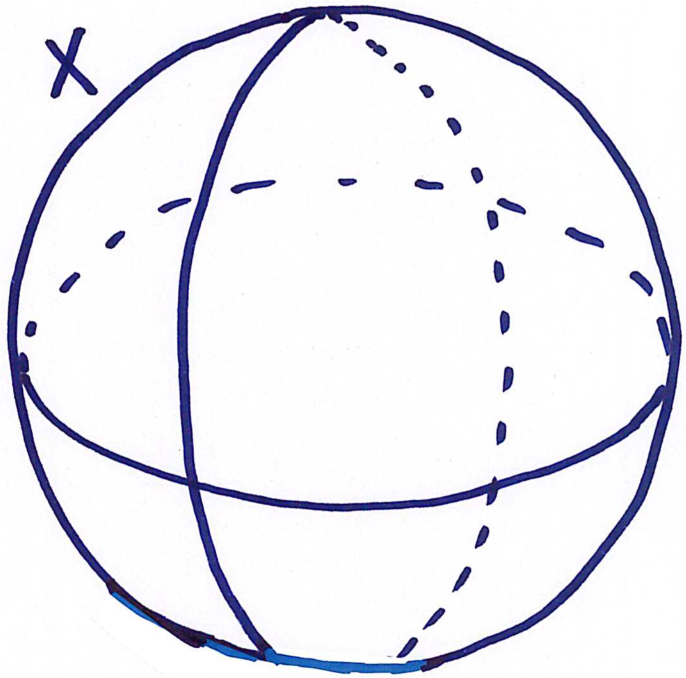
F =

V–E+F =

Euler Characteristic & Genus of Two-Manifolds

Let X be a surface with Euler characteristic denoted $e(X)$ and genus denoted $g(X)$. Then

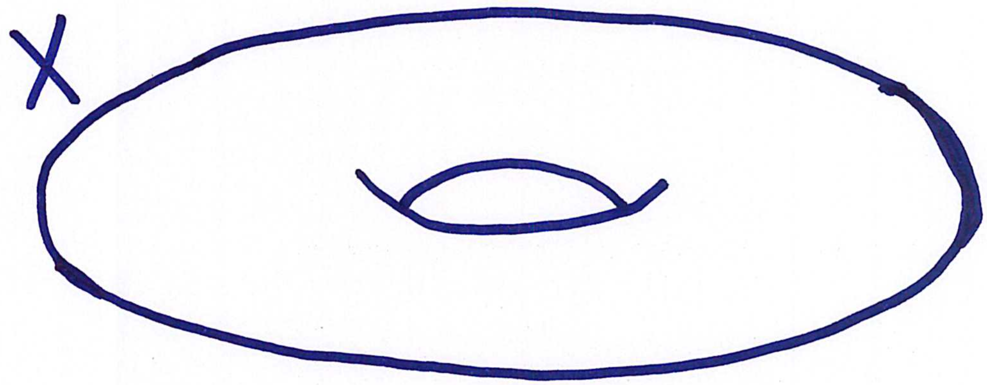
Euler Characteristic & Genus Example: Sphere



Genus:

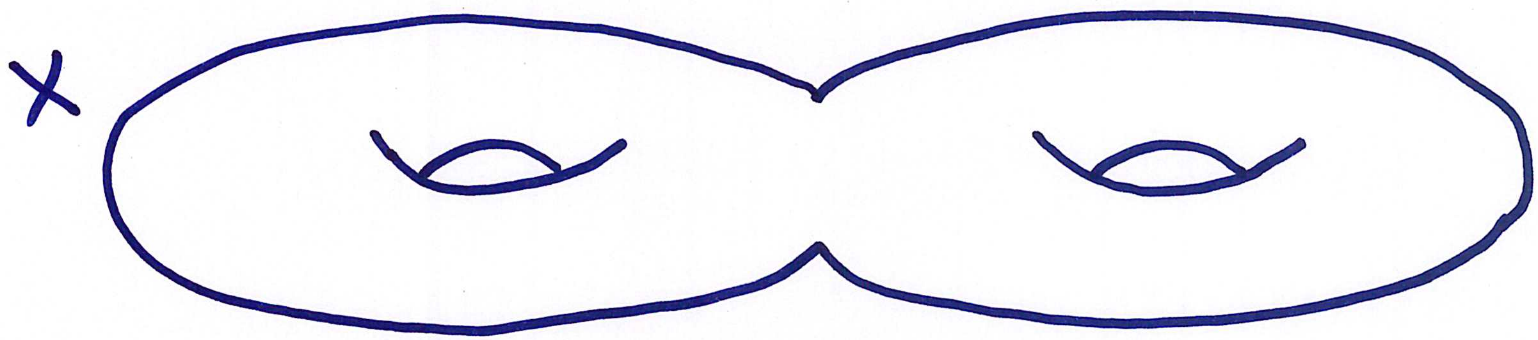
$g(X) = 0$

Euler Characteristic & Genus Example: Torus



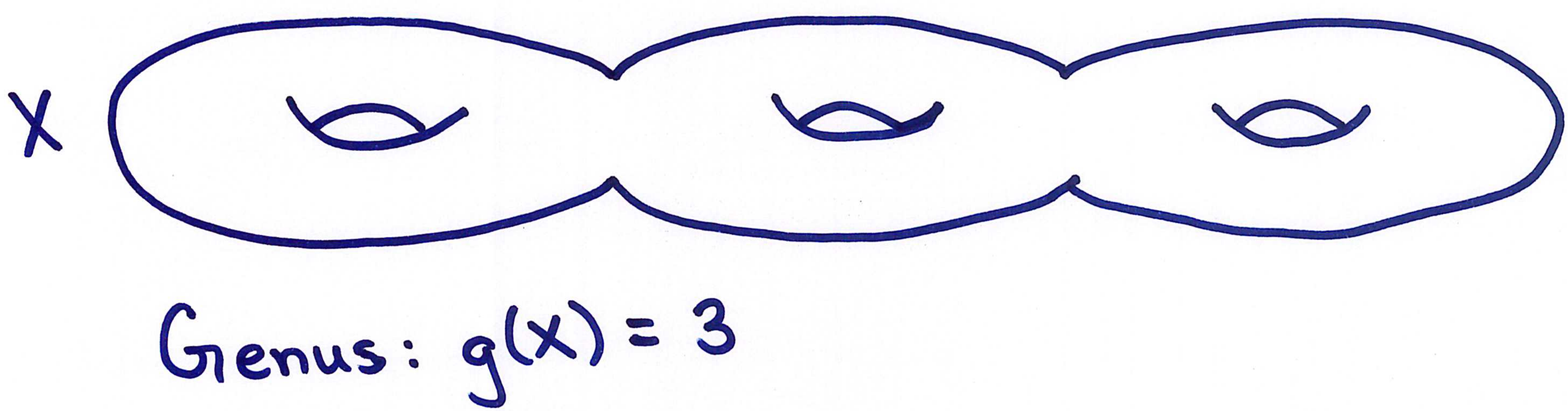
Genus: $g(X) = 1$

Euler Characteristic & Genus Example: Connected Sum of Two Tori

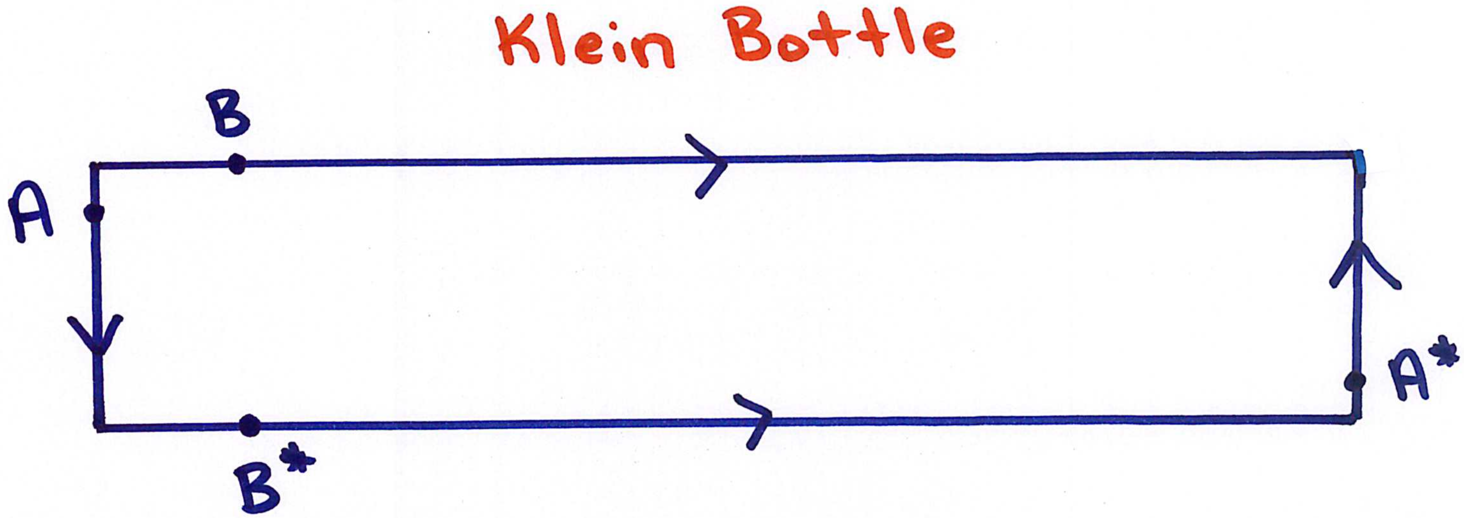


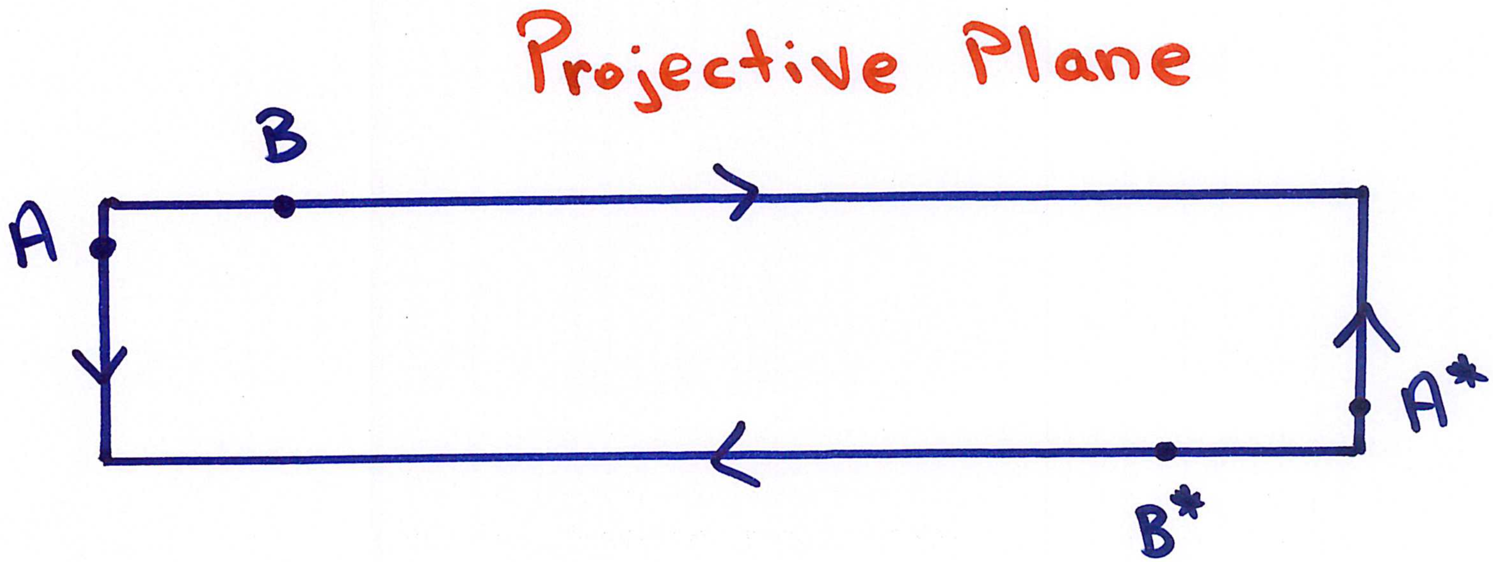
Genus: $g(X) = 2$

Euler Characteristic & Genus Example: Connected Sum of Three Tori



Some Non-Orientable Two-Manifolds





Classification of Orientable Two-Manifolds

Every orientable two-manifold is *homotopic* to:

- a sphere;
- a torus; OR
- a connected sum of (any finite number of) tori.

Classification of Non-Orientable Two-Manifolds

Every non-orientable two-manifold is *homotopic* to:

- a projective plane; OR
- a connected sum of (any finite number of) projective planes.

QUESTIONS???