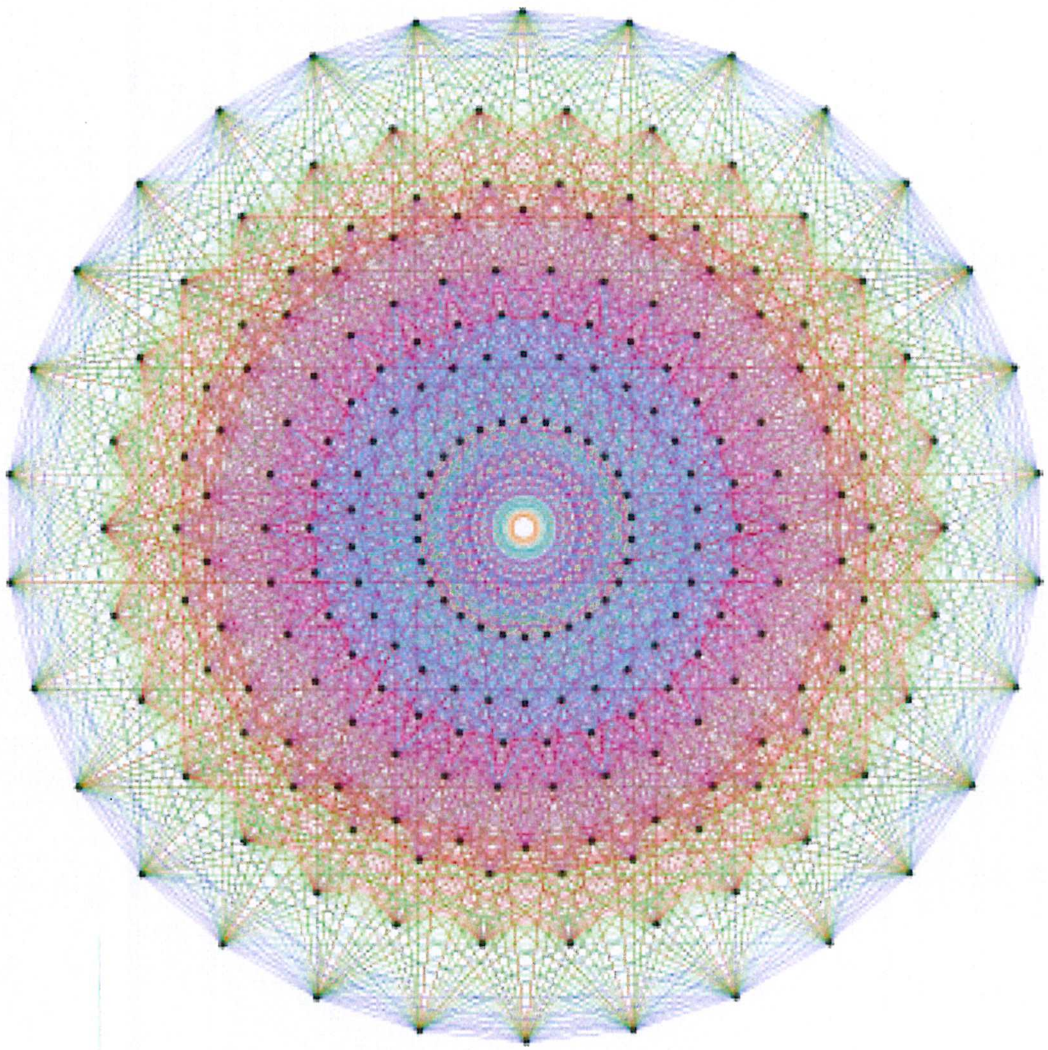
Symmetries



Transformations - Definitions (page 33 of text)

- A *transformation* of the points in the plane is a

- If no two points are moved to a single position, then we say that the transformation is

- A transformation is *onto* if all the positions in the plane are achieved by some points in the rearrangement.

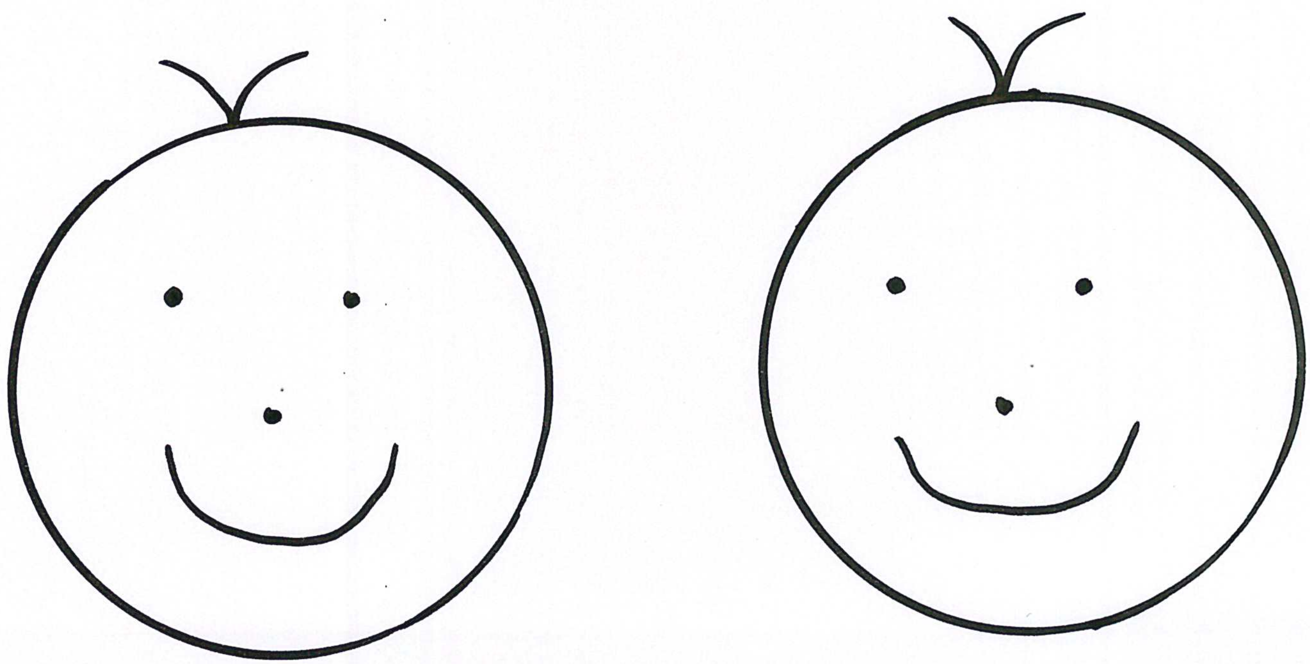
- A *bijection* is a transformation that is both

Symmetries Definition (page 33 of text)

- A transformation is *rigid* if it preserves
- Rigid transformations are called

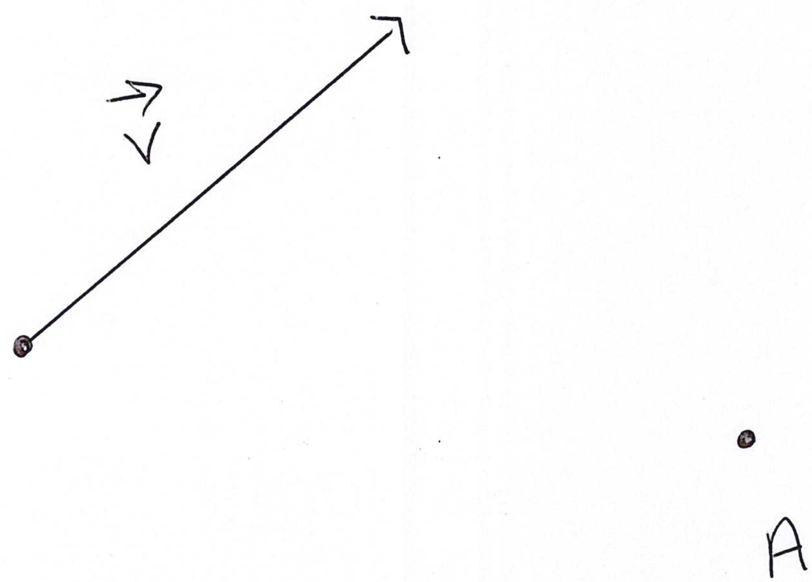
Translations

A *translation* is defined by a vector $\overrightarrow{v}$ and is denoted $f = trans(\overrightarrow{v})$.



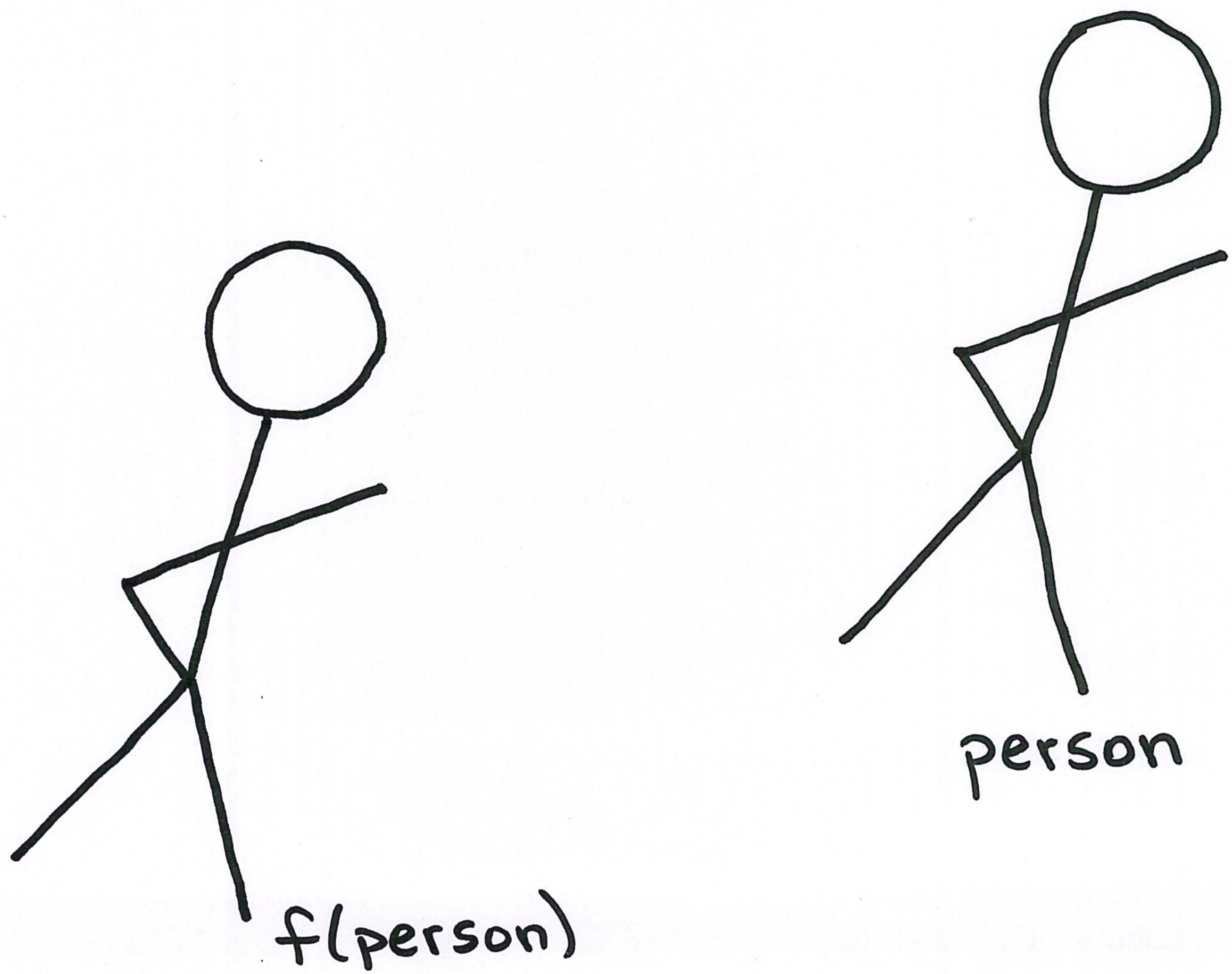
Example

Find the image of A under the symmetry $f = trans(\overrightarrow{v})$



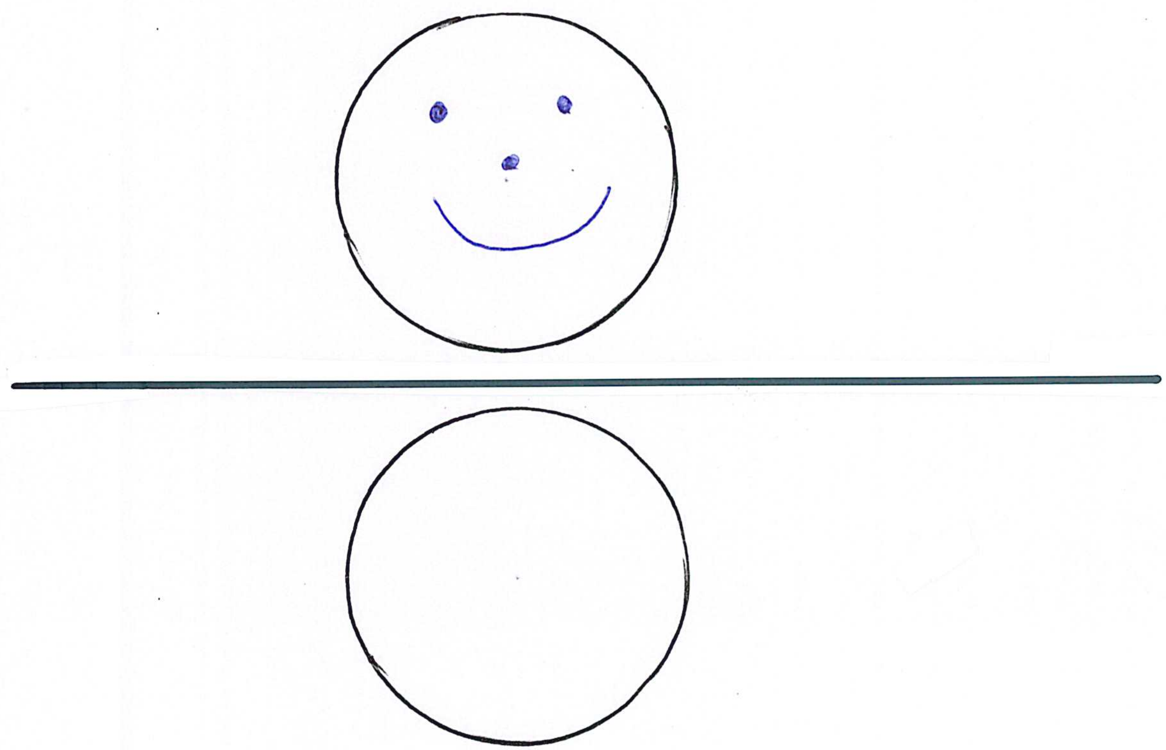
Example

Find the vector of translation $\overrightarrow{v}$ of the symmetry $f = trans(\overrightarrow{v})$.



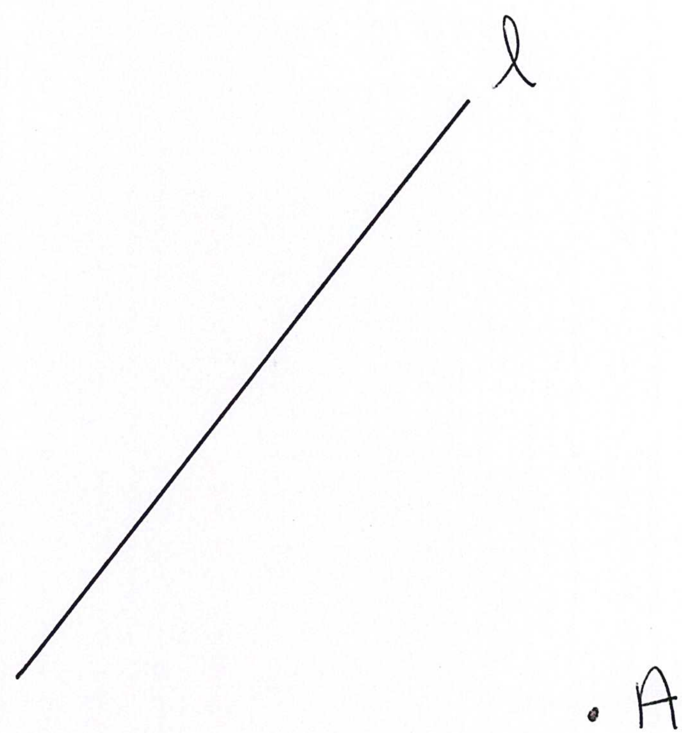
Reflections

A *reflection* is defined by a line ℓ and is denoted by $f = \mathit{refl}(\ell)$.



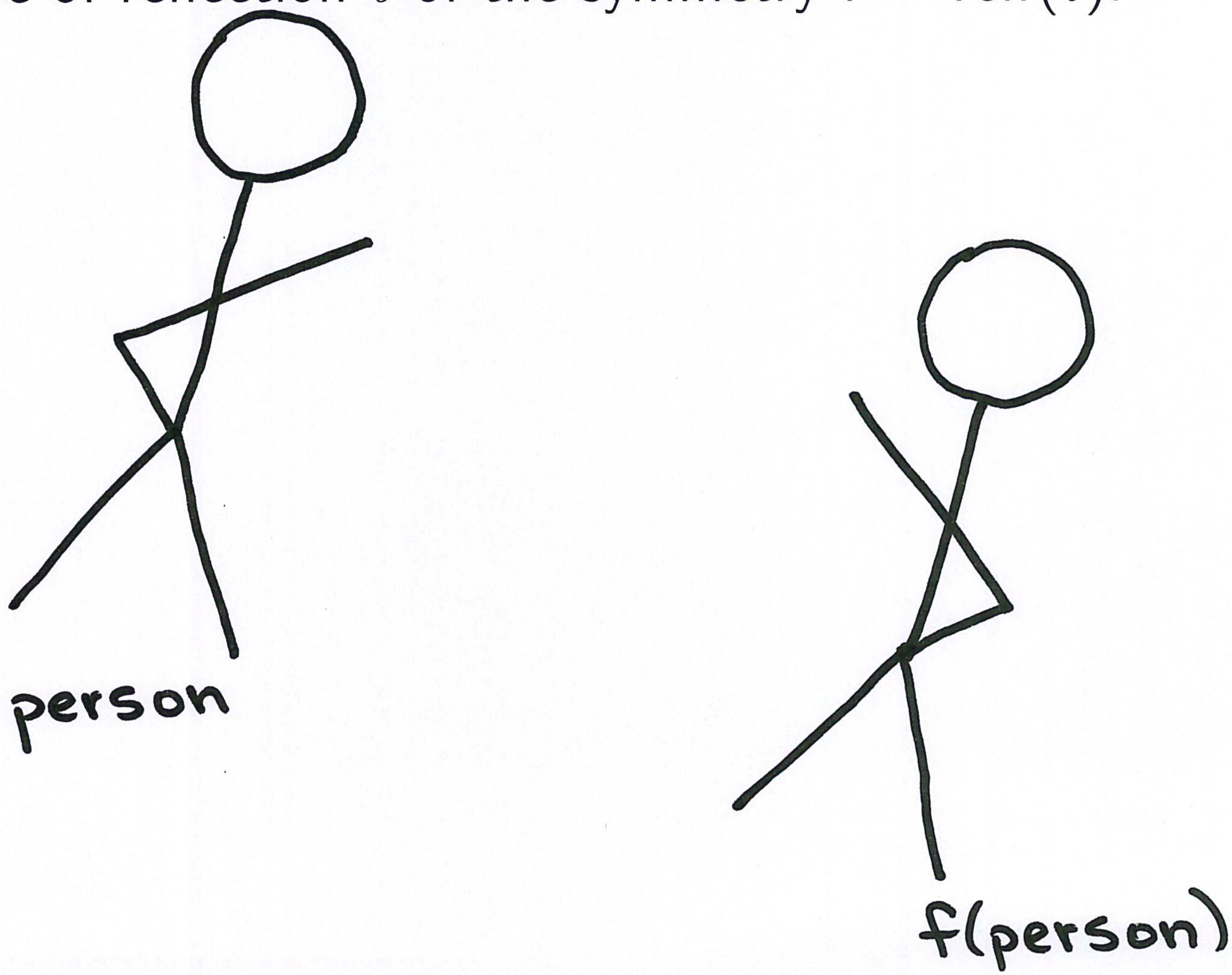
Example

Find the image of A under the symmetry $f = refl(\ell)$.



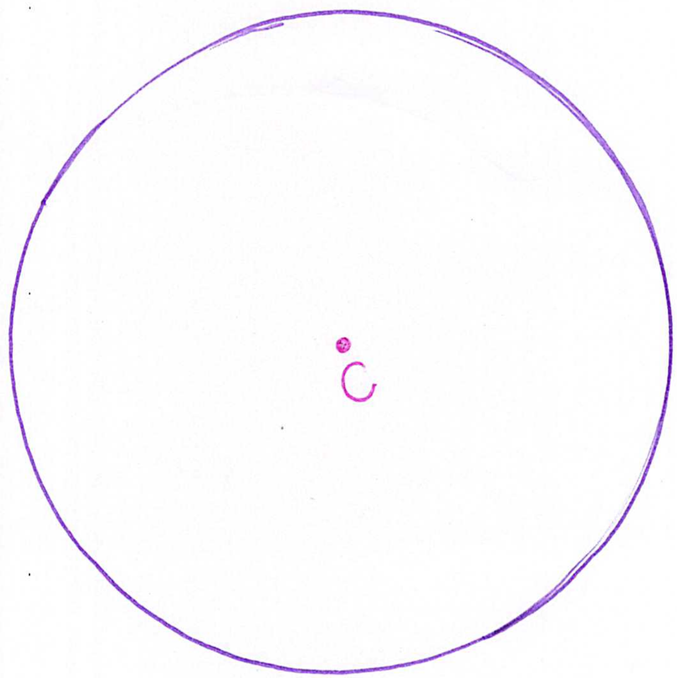
Example

Find the line of reflection ℓ of the symmetry $f = \mathit{refl}(\ell)$.



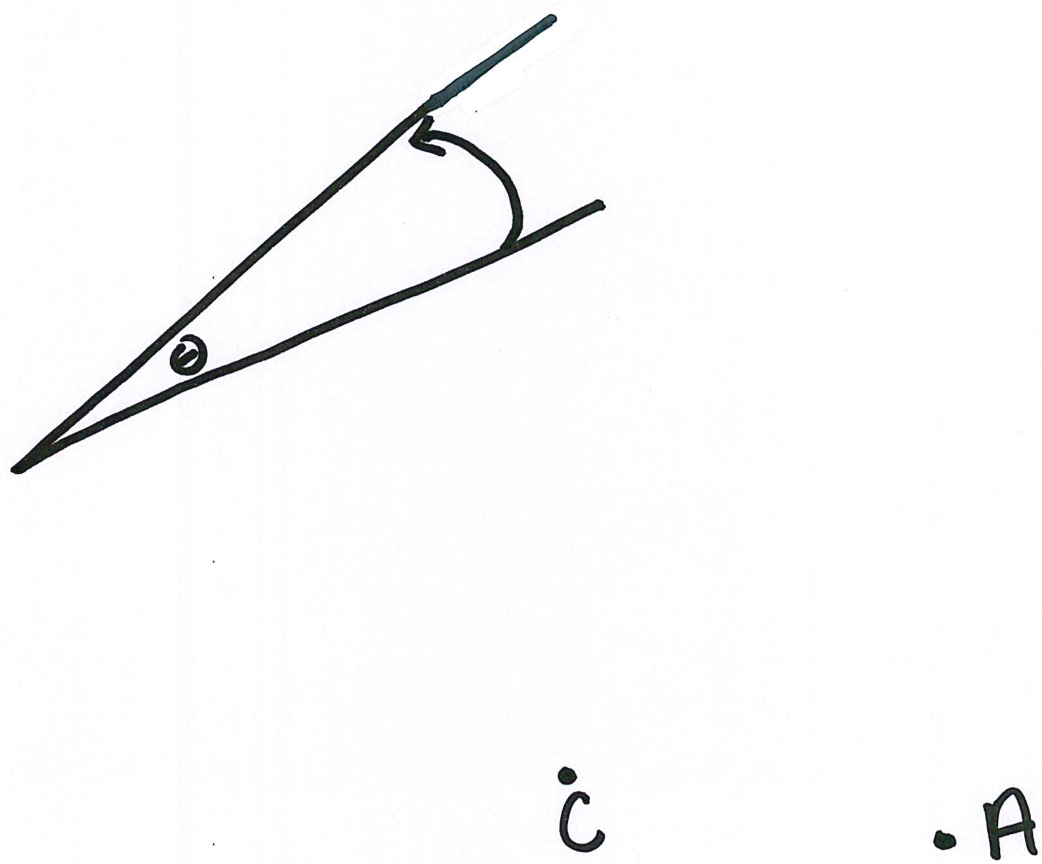
Rotations

A *rotation* is defined by an angle θ and a centre C of a circle, denoted $f = rot(C, \theta)$.

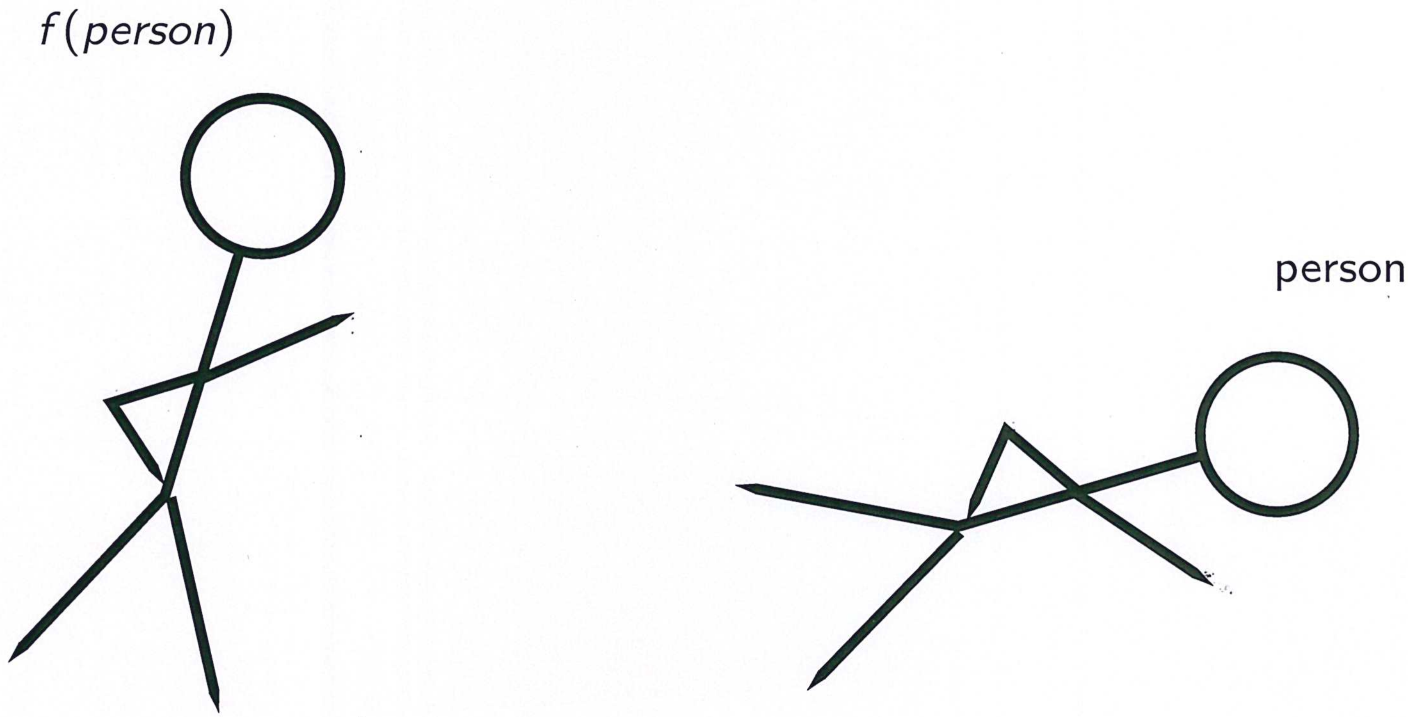


Example

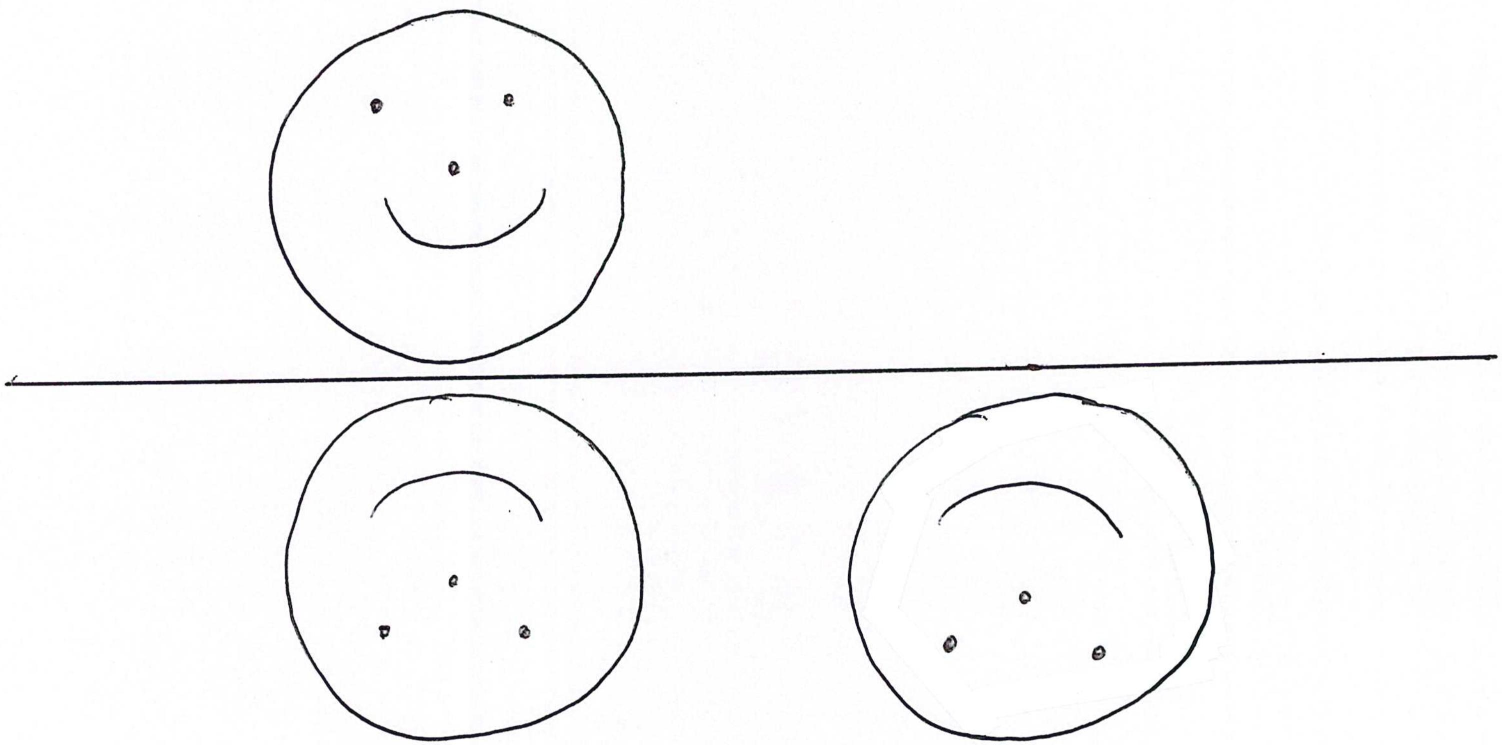
Find the image of A under the symmetry $f = rot(C, \theta)$.



Find the center and angle of the symmetry $f = rot(c, \theta)$



Example



The Classification Theorem For Plane Symmetries

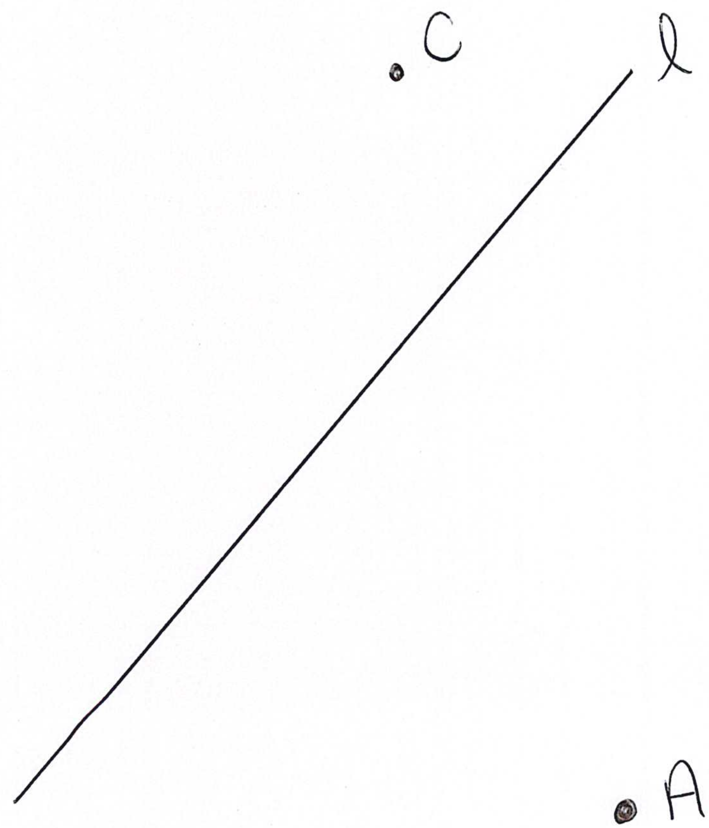
Theorem

Every symmetry of the plane is either:

- *a composition of a translation followed by a rotation; or*
- *a composition of a translation followed by a reflection.*

Example

Find the image of A under the composition of the symmetries $f_1 = \mathit{refl}(\ell)$ followed by $f_2 = \mathit{rot}(C, 60^\circ)$.



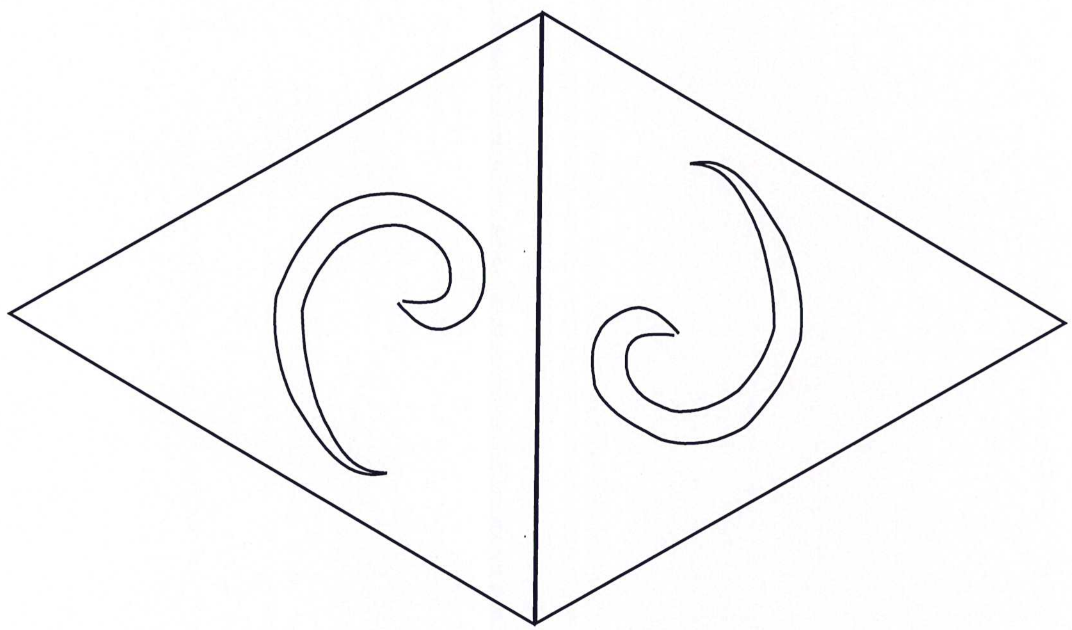
Definitions (page 55 & 56 of text)

- Given an object O in the plane, a *symmetry of the object* O is a symmetry of the plane that rearranges the points of O within the points of O such that every position in the object is attained by some point following the rearrangement.

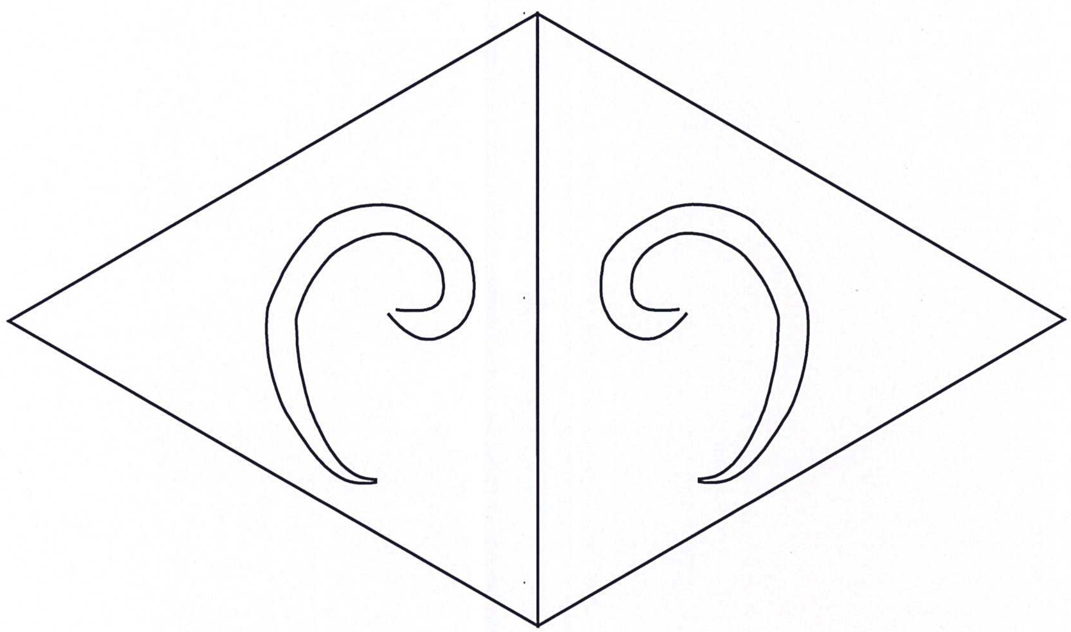
  Note: We can think of this as a symmetry under which the object

- The set of all symmetries of an object is called the

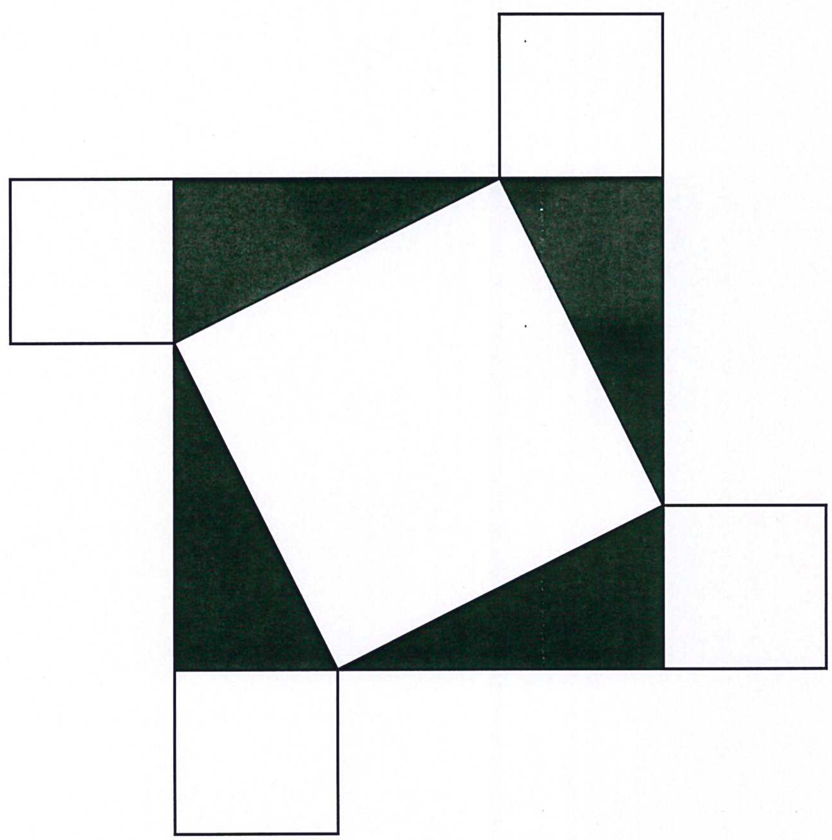
Find the group of symmetries of the following object



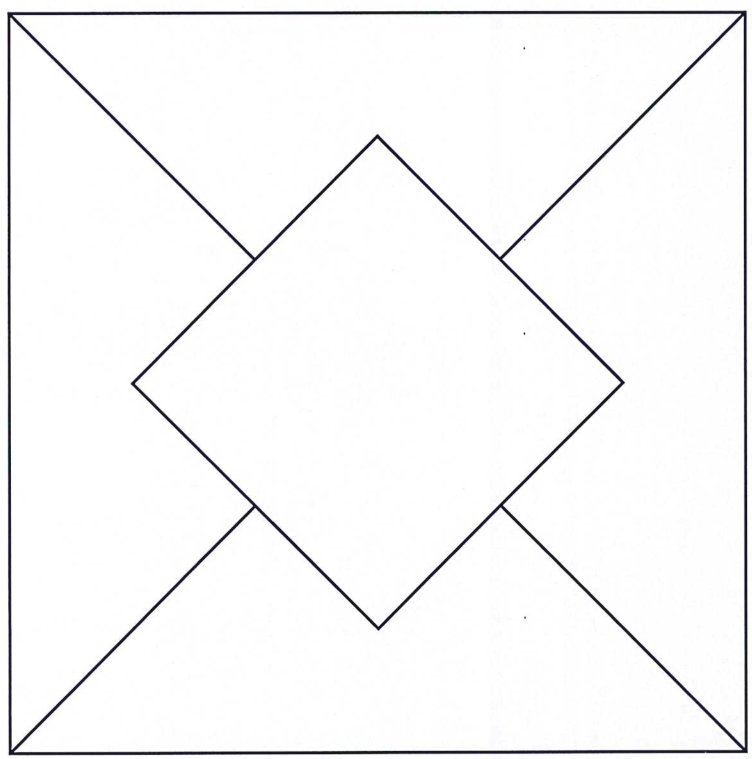
Find the group of symmetries of the following object



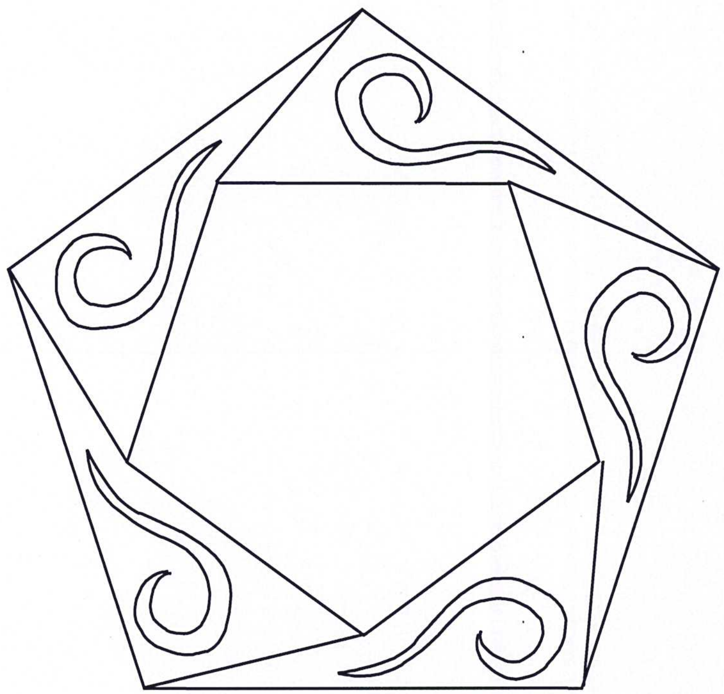
Find the group of symmetries of the following object



Find the group of symmetries of the following object



Find the group of symmetries of the following object



QUESTIONS???