Math 1020 Math in Art
Midterm Exam
February 28, 2008

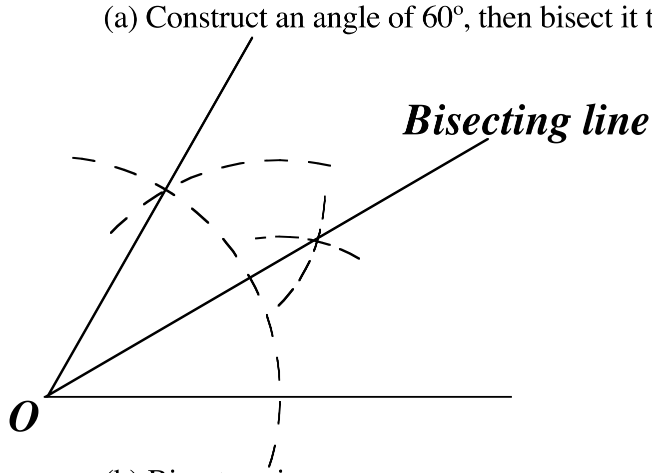
1.	*max=7*	
2.	*max=6*	
3.	*max=9*	
4.	*max=11*	
5.	*max=7*	

Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

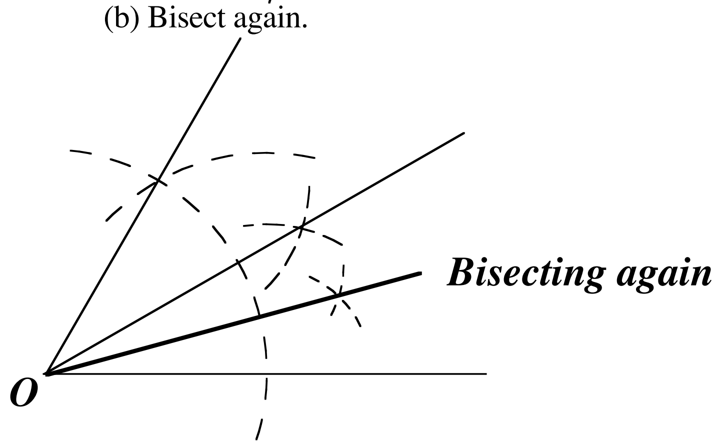
[7 points] 1. (a) Construct (using an unmarked ruler and a compass) an angle of 30º with a corner at O and over the given semi-line.
(b) Bisect the angle constructed in part (a) of this exercise.

Solution. Sketch of the solution of 1(a) is given in the illustration to the left; part (b) is sketched to the right.

(a) Construct an angle of 60º, then bisect it to get an angle of 30º.



(b) Bisect again.



[6 points] 2. Construct one of the two golden cuts of the line segment given below.

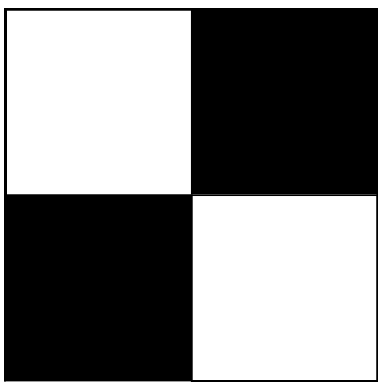
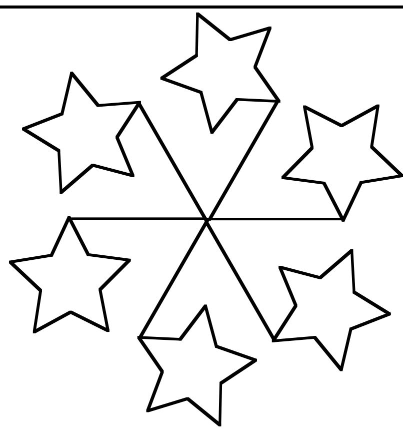
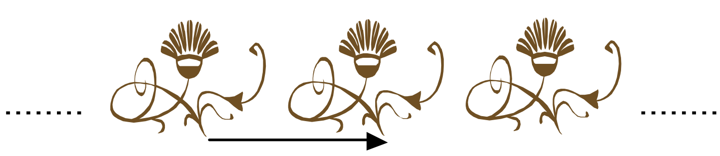
Solution. This is given in the textbook (and in the class notes)

[9 points] 3. (a) Construct a golden rectangle with the line segment given below representing the base of the rectangle (so that it is one of the larger sides of the rectangle).

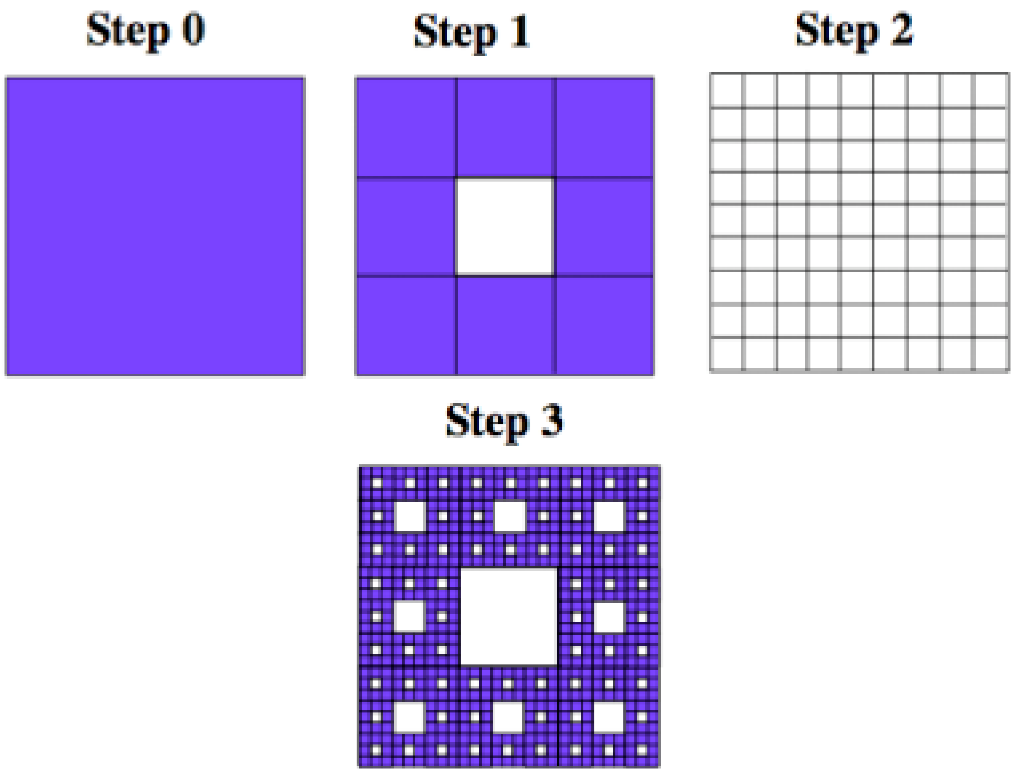
(b) Construct a golden spiral within the golden rectangle shown below.

Solution. Both (a) and (b) are solved in the textbook and during class lectures.

[11 points] **4.** Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation, drawing **precisely** at least one of them. [In all three cases the object is defined by the (black or gray) coloured points.]

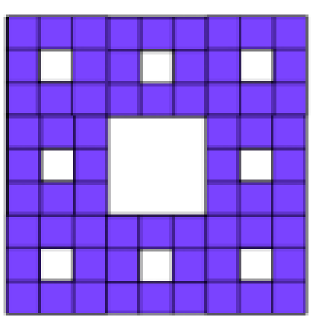
OBECT	THE GROUP OF SYMMETRIES
	Denote by l_1 and l_2 the lines through the diagonals of the largest square, and denote by *O* their intersection. {*id*, rot(O,180º), ref(l_1), ref(l_2)}
	Denote by *O* the center of the object. {*id*, rot(*O*,60º), rot(*O*,120º), rot(*O*,180º), rot(*O*,240º), rot(*O*,300º)}
 [This is a Frieze pattern and it extends without end both to the left and to the right.	Let ***v*** stands for the vector shown to the left. {*id*, trans(***v***), trans(2***v***), trans(3***v***),….., trans(*n**v***),…., trans(-***v***), trans(-2***v***), trans(-3***v***),, …, trans(*n**v***),…}

[7 points] 5. (a) You are shown some of the first few steps of a procedure generating fractal: the initial figure (Step 0), the figure we get after 1 iteration (Step 1), and the figure we get after 3 iterations (Step 3). Shade with a pencil within the grid shown below Step 2 the object we get after performing 2 iterations.



(b) After iterating (repeating the above steps) infinitely many steps we get a fractal F. That fractal is self-similar with respect to a central similarity centered at a point A and with a stretching factor α. Indicate in the figure shown below Step 3 above the position of the point A and write down the value of α. (You only need to indicate where A is, and fill the blank in $\alpha =$ ______ .)

Solution. **(a)**



(b) Choose A to be any corner of the largest square; $\alpha = \frac{1}{3}$. (Any of $\alpha = \frac{1}{3^n}$, n=1, 2, 3,… would do!)