Math 1020 Math in Art
Midterm Exam, October 20, 2014

Name: ______________________________ **Student Number:** ______________

1.	*max=8*	
2.	*max=10*	
3.	*max=9*	
4.	*max=13*	
5.	*max=10*	

Total=50

Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

[9 points] 1. **(a) [4]** Construct an angle of 45°.

Step 1. Construct two perpendicular line; the angle between them is 90°.
Step 2. Bisect the angle of 90° to get and angle of 45°.

(b) [4] Subdivide the line segment *AB* shown below into three ***equal*** parts by constructing two points between *A* and *B*.

~~The is~~ done on page 11, textbook; we also did essentially the same thing in class.

[10 points] 2.

(a) [7] Recall that a line segment AB has two golden cuts: one closer to A, the other closer to B. Construct the golden cut C of the line segment AB (as given below) such that C is closer to A than to B.

Done in class and in the book; you could do directly, or first the golden cut that is closer to B, then carry it over with a compass to the golden cut closet to A.

(b) [3] Recall that in a golden rectangle the ratio of the base with respect to the height is the golden section ϕ. Construct a rectangle with its base equal to the line segment AB given below, and the ratio of its base over its height equal to 2ϕ. You may carry over (with a compass) any needed line segments constructed in part (a).

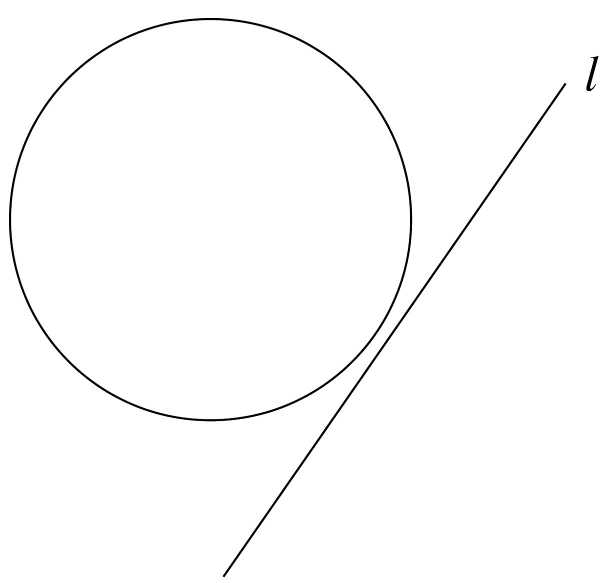
Step 1. Construct a golden rectangle over AB; done in class and in the text; you could have used part (a) and transferred some line segments from there to here.
Step 2. Subdivide the height into half; the twice shorter rectangle is the one we want.

[9 points] 3. **(a) [4]** Let C be a fixed point (choose it somewhere in the space below this paragraph). It is true that the composition $rot(C,25^{\circ}) \circ rot(C,-19^{\circ})$ is a basic symmetry. Which one? If your answer is a rotation, then you specify the center of the rotation and the angle of the rotation (in degrees). If your answer is a translation, sketch the vector of translation. If your answer is a reflection, sketch the line of the reflection.

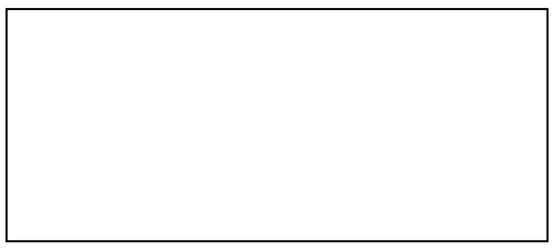
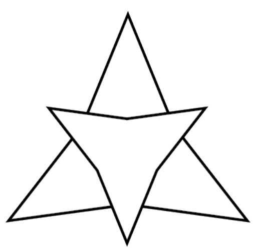
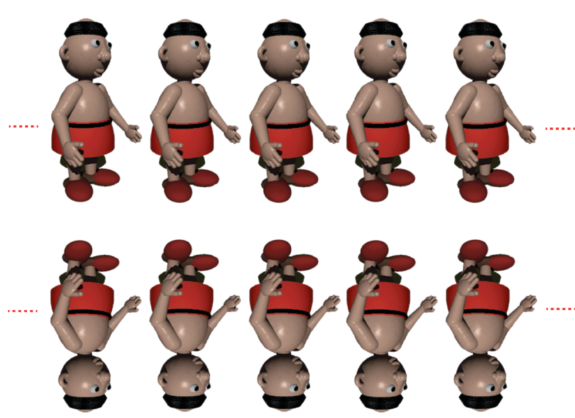
Answer: $rot(C,6^{\circ})$

(b) [5] Reflect the circle shown below with respect to the line l. Construct the image-circle precisely. [Hint: the center of the circle is important; construct it first!]

Step 1. Find the center of the circle (done in class and in the book). Denote it by, say, C.
Step 2. Reflect the point C with respect to l (done in class and in the book). Get a point C'.
Step 3: measure with a compass the radius of the given circle; draw a circle with that radius centered at C'.



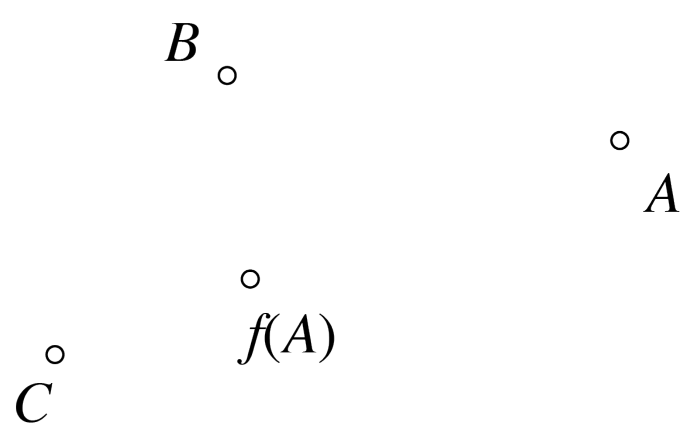
[13 points] 4. Find the group of symmetries of each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If you use translations, describe the vectors of the translation, drawing **precisely** at least one of them. [The objects are defined by the black points.]

OBJECT	THE GROUP OF SYMMETRIES
	(a) [3] Solution: $\{id, rot(C,180^\circ), ref_m, ref_n\}$ where C is the center of the rectangle, m is the vertical line through C, n is the horizontal line through C. C, m, n should have been shown.
	(b) [4] Solution: $\{id, rot(C,120^\circ), rot(C,240^\circ), ref_l, ref_m, ref_n\}$ where C is the center of the object, l , m, n are the obvious lines through C. C, l, m, n should have been shown.
 [This is a Frieze pattern and it extends without end both to the left and to the right.]	(c) [6] Solution: $\{id, tran_{\mathbf{v}}, tran_{2\mathbf{v}}, \ldots, tran_{-\mathbf{v}}, tran_{-2\mathbf{v}}, \ldots, g \circ tran_{\mathbf{v}}, g \circ tran_{2\mathbf{v}}, \ldots, g \circ tran_{-\mathbf{v}}, g \circ tran_{-2\mathbf{v}}, \ldots\}$ where **v** is the vector from the tip of the head of one character to the tip of the head of the next one (**v** should have been drawn in the picture) and where g is the reflection with respect to the horizontal line through the middle of the patter (this line should have been shown too).

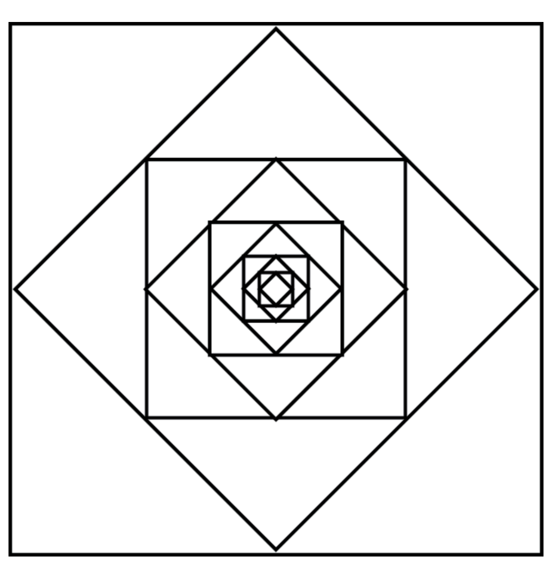
[10 points] 5.

(a) [6] Let f be the central similarity centered at the point C (shown below). We are also given the points A and B, and the point $f(A)$ obtained by applying f to A. Construct the point $f(B)$ obtained by applying f to B.

Step 1. Join C and B
Step 2. Connect A and B
Step 3. Construct the line through f(A) that is parallel to AB. Where this line intersects the line segment CB is the point f(B).



(b) [4] In the figure below every square except the largest joins the middle points of the sides of the next largest square. This procedure goes without end, and the final outcome is a fractal called F. Confirm that the result is indeed a fractal: find a proper central similarity f that sends F within itself. You get the full mark if you indicate in the figure the center of your central similarity f, and write down the value of the stretching factor α of your central similarity.



Solution: the center of the central similarity that will do the work is in the center of this figure; the stretching factor that will do is ½.