Math 1020 Math in Art
Midterm Exam, October 14, 2008

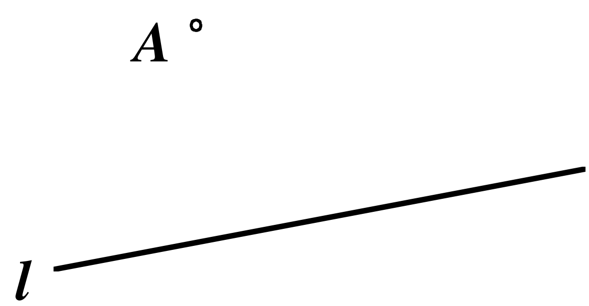
Name: ______________________________ Student Number: ______________

1.	*max=10*	
2.	*max=10*	
3.	*max=6*	
4.	*max=14*	
5.	*max=10*	

Total=50

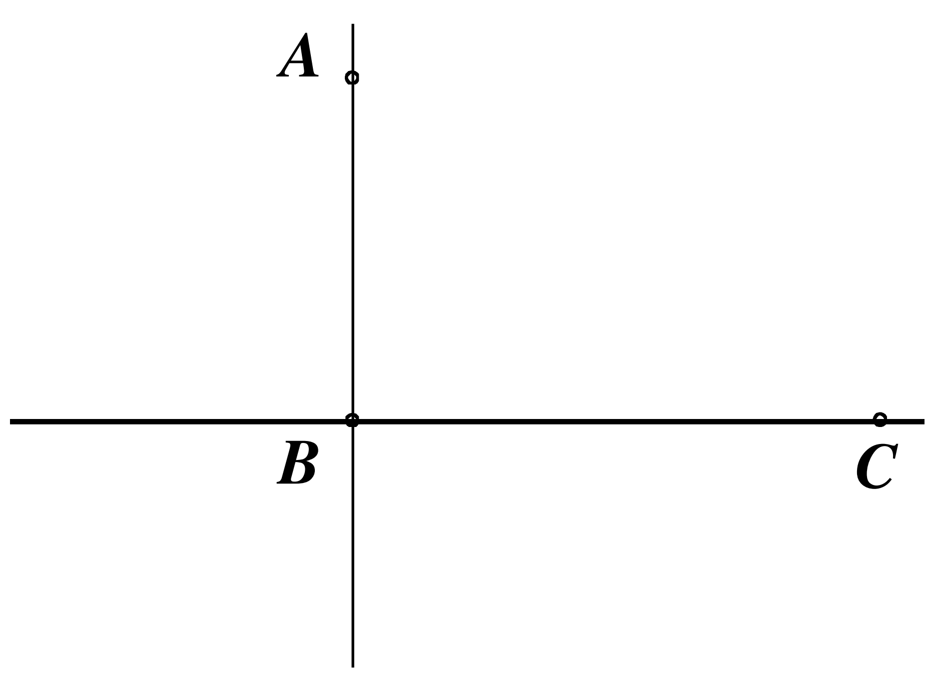
Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

[10 points] 1. (a) Construct the line passing through the given point *A* and that is perpendicular (normal) to the given line *l* (see the picture below).



Solution. This is construction #2, pages 7-8 in the textbook.

(b) The two lines shown in the next picture are perpendicular. Subdivide (using, of course, only an unmarked ruler and a compass) the angle $\angle ABC$ into three equal angles.

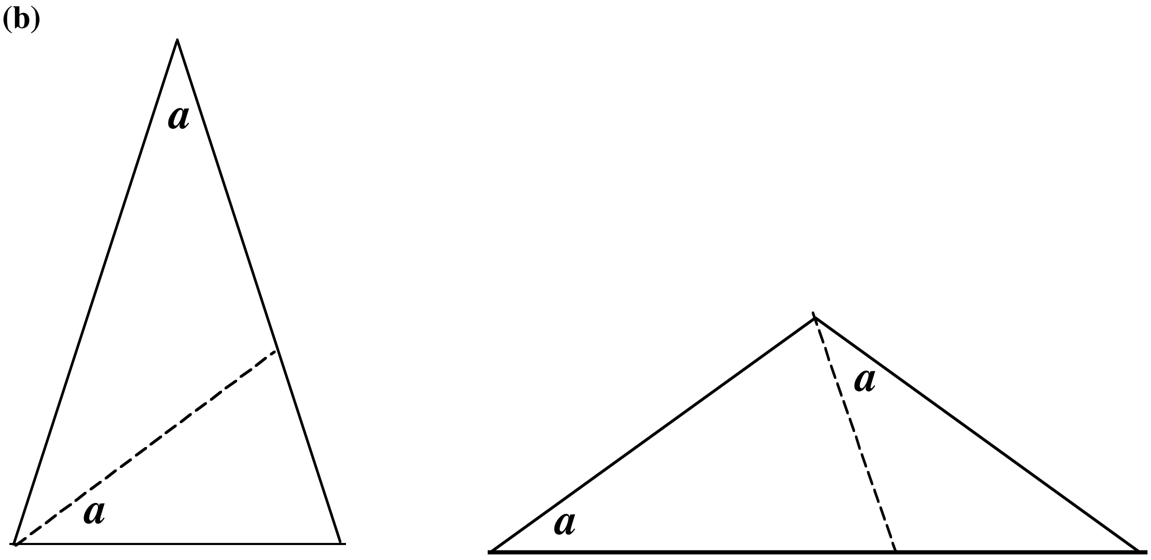


Solution. First construct an equilateral triangle over the line segment BC (done in class), then subdivide the interior angle at B into two equal angles (done in class).

[10 points] **2.** **(a)** Construct an obtuse golden triangle over the line segment *AB* (so, the line segment *AB* should be the basis of the obtuse golden triangle you construct).

(b) Subdivide the golden triangle you construct in part (a) into two smaller triangles, one of them an acute golden triangle, the other an obtuse golden triangle. Explain clearly, but in no more than two sentences, why these two smaller triangles are golden.

Solution. Part (a) was done in class (several times; check your notes).
(b)



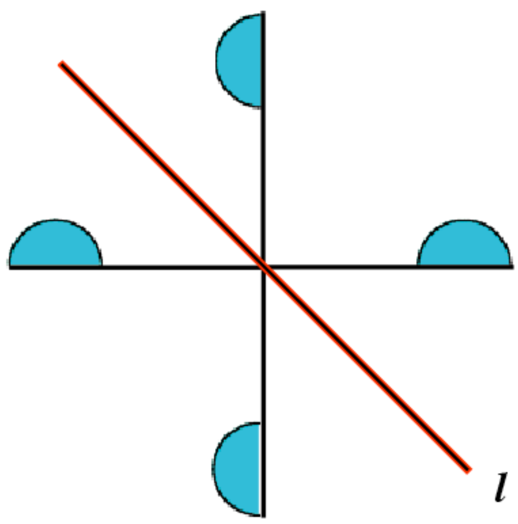
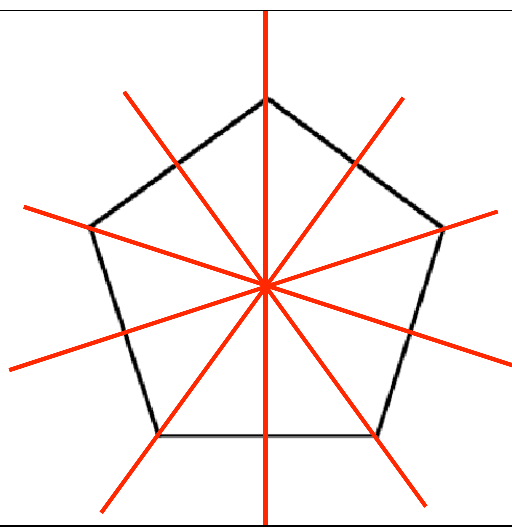
The picture to the right is needed here; it shows an obtuse golden triangle. Simply duplicate the angle *a* as indicated. The angles of the two smaller triangles make them golden.

[6 points] **3.** **(a)** State precisely the definition of the sequence of Fibonacci numbers.

(b) The 27th Fibonacci number is 196418, and the 26th Fibonacci number is 121393. Find the 25th Fibonacci number.

(a) Check the book or the notes.
(b) $f_{27} = f_{26} + f_{25}$, so that $196418 = 121393 + f_{25}$. From there we find $196418 - 121393 = f_{25}$, or $f_{25} = 75025$.

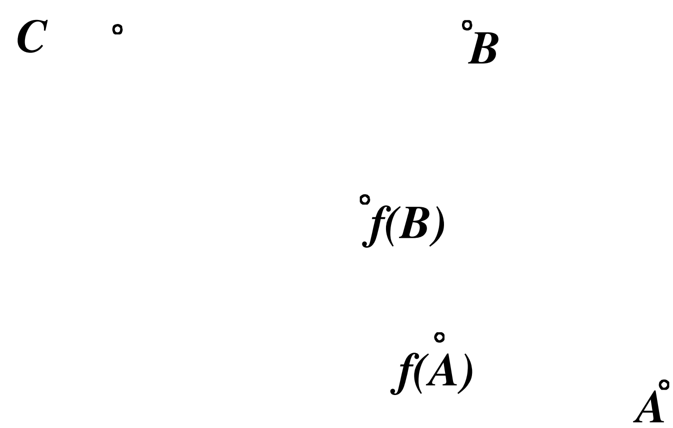
[14 points] **4.** Find the group of symmetries for each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If there are translational symmetries show or describe the vectors of translation, drawing **precisely** at least one of them. [In all three cases the object is defined by the (black or gray) coloured points.]

OBECT	THE GROUP OF SYMMETRIES
	{identity, reflection with respect to the red line as shown}
	Five reflections with respect to the red lines shown to the left, and five rotations as follows (where O is the center of the pentagon): $rot(O,0^0) = identity$, $rot(O,72^0)$, $rot(O,144^0)$, $rot(O,216^0)$, $rot(O,288^0)$.
 [This is a Frieze pattern and it extends without end both to the left and to the right.	Denote by ***v*** the red vector is shown. Then the group of symmetries of this Frieze pattern consists of the identity and the following translations: $\{trans_v, trans_{2v}, trans_{3v}, ...\}$ together with $\{trans_{-v}, trans_{-2v}, trans_{-3v}, ...\}$.

[10 points] 5. We know that the point $f(A)$ is obtained from the point A by applying to it a central similarity f, and the point $f(B)$ is obtained from the point B by applying to it the same central similarity f (see the picture below).

(a) Construct the center O of the central similarity f.

(b) Construct the image $f(C)$ of the given point C under the central similarity f.



Solution. This question was done completely in the class; check your notes.