Brief Solutions

UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH/FA 1020 Math in Art
Midterm Exam
October 25, 2017

LAST NAME: (Print in ink) ____________________

FIRST NAME: (Print in ink) ____________________

STUDENT NUMBER: (in ink) ____________________

SIGNATURE: (in ink) ____________________
(I understand that cheating is a serious offense)

☐ A01 Susan Cooper

☐ A02 Sasho Kalajdzievski

Question	Points	Score
1	7	
2	10	
3	6	
4	10	
5	13	
6	9	
Total:	55	

INSTRUCTIONS TO STUDENTS:

Fill in all the information above.

No texts, notes, cell phones or other aids are permitted.

Simple calculators are permitted, as is a ruler and a compass.

Show your work clearly for full marks.

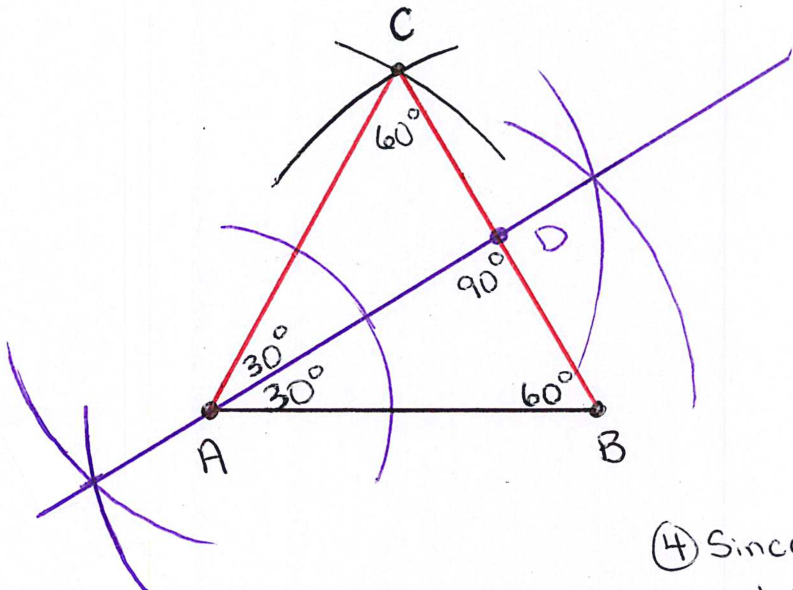
This exam has a title page, 6 pages of questions and 1 blank page at the end for rough work.
Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 55.

Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the reverse side of the page, but clearly indicate that your work is continued there.

Important: **"Construct" means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do NOT erase them). Use words to describe BRIEFLY what you have done.**

[7 points] 1. (a) [4] Construct a triangle with interior angles of 90°, 30° and 60°.



① Construct a line segment AB.

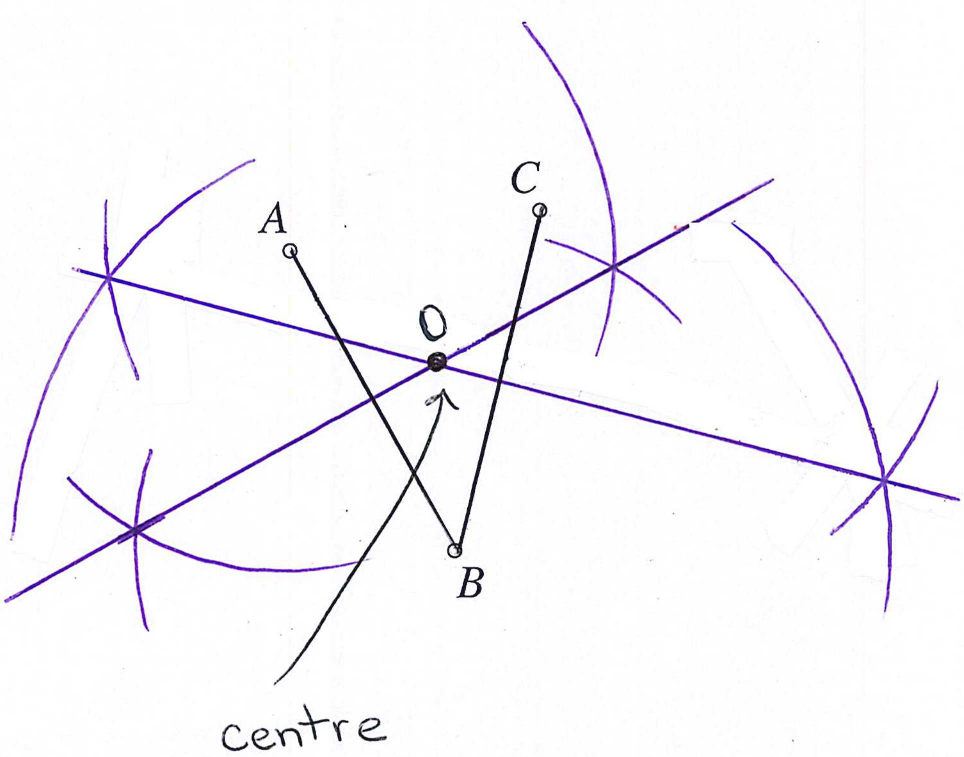
② Construct equilateral triangle Δ ABC. Every interior angle is 60°

③ Bisect any angle in ΔABC to get a 30° angle.

④ Since angles sum to 180° in a triangle, angle ADB is 90°

⑤ Triangle ΔABD has the desired angles.

(b) [3] The points *A*, *B* and *C* (shown below) all lie on a single circle. Construct the center of that circle.



This is Construction 5 of Section 1.2 of the text (pg 9)

① Draw AB and BC

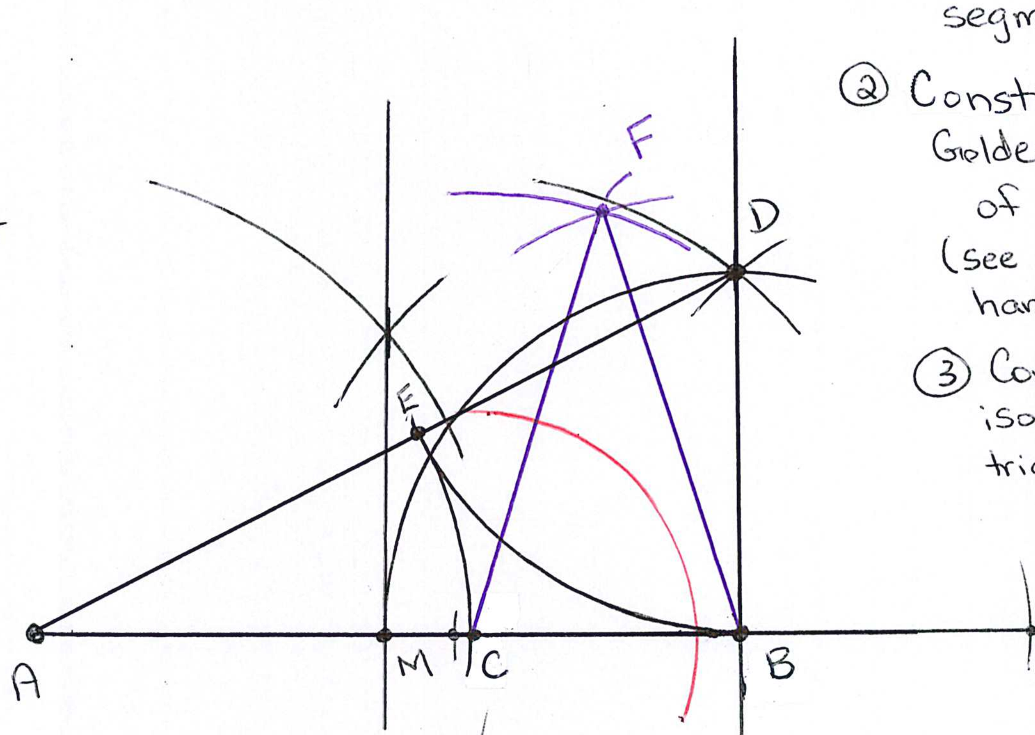
② Bisect line segments AB and BC

③ The intersection point O of the bisecting lines from step 2 is the centre of the circle (with radius OA = OB = OC)

[10 points] 2.

(a) **[5]** Construct any acute golden triangle.

* See pg 19 of text



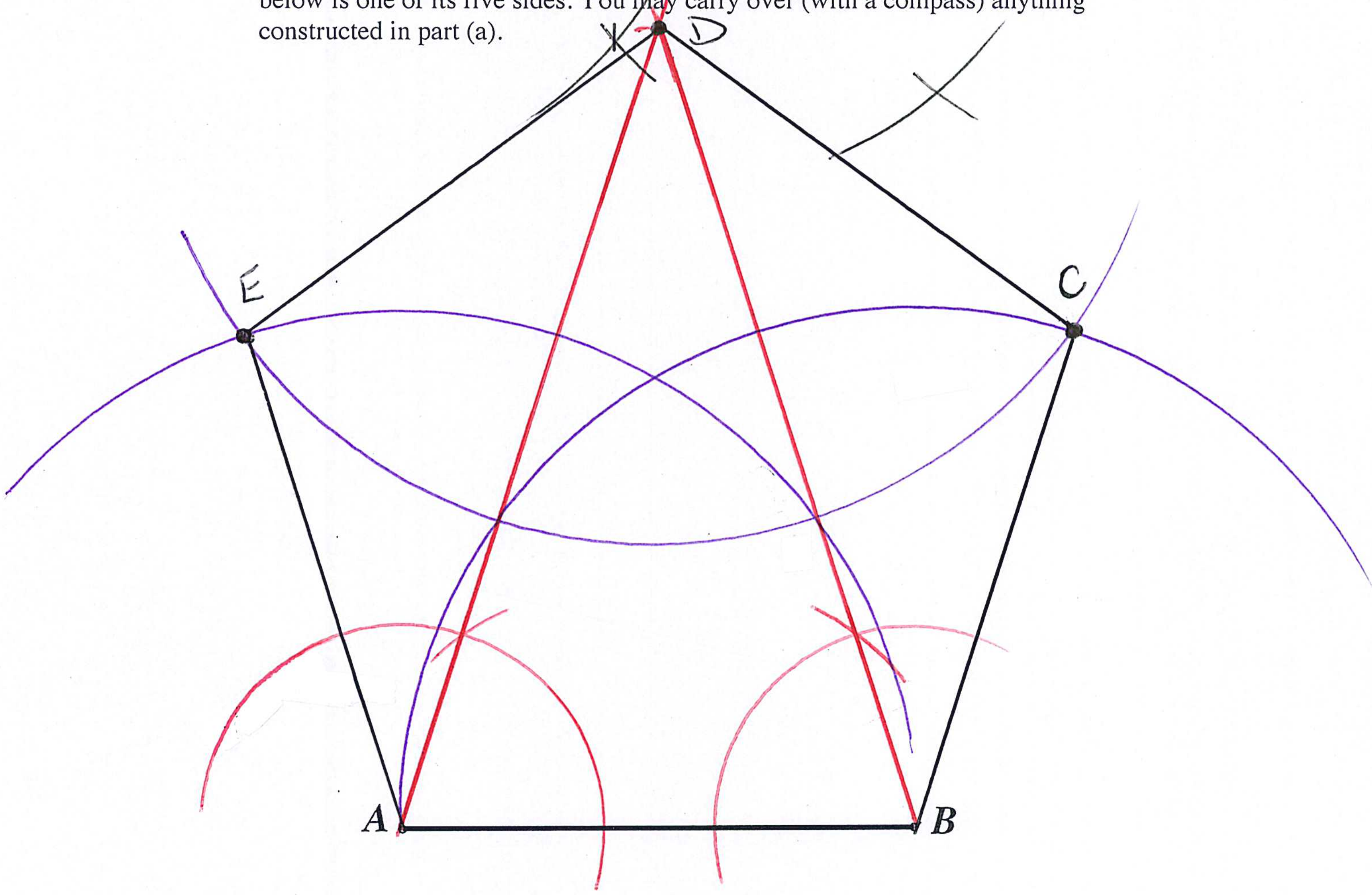
① Draw line segment AB

② Construct Golden Cut C of AB. (see class handout)

③ Construct isosceles triangle over CB with other 2 sides having length AC.

④ Triangle ΔCFB is an acute golden triangle.

(b) **[5]** Construct a regular pentagon so that the line segment *AB* shown below is one of its five sides. You may carry over (with a compass) anything constructed in part (a).



① Copy angle FCB (72°) to A and B to construct an acute golden triangle over AB. (in red)

② Draw circles centred at A, B, D each of radius AB

③ Find intersection points C and E

④ Connect points appropriately to obtain a regular pentagon

* See pg 22 of text

[6 points] 3. (a) [2] What are Fibonacci numbers? (Write down a precise definition.)

A sequence of numbers $f_1, f_2, f_3, f_4, \ldots$

such that

$$f_1 = 1$$

$$f_2 = 1$$

and $f_n = f_{n-1} + f_{n-2}$ for $n \geq 3$.

(b) [4] In this problem f_{20}, f_{21}, f_{22} are Fibonacci numbers. If $2f_{20} = 13530$ and $2f_{22} = 35422$, compute f_{21}. Show your work! (You get no marks if your answer is not justified.)

$$f_{22} = f_{21} + f_{20}$$

Now $2f_{22} = 35422 \Rightarrow f_{22} = 17711$

$2f_{20} = 13530 \Rightarrow f_{20} = 6765$

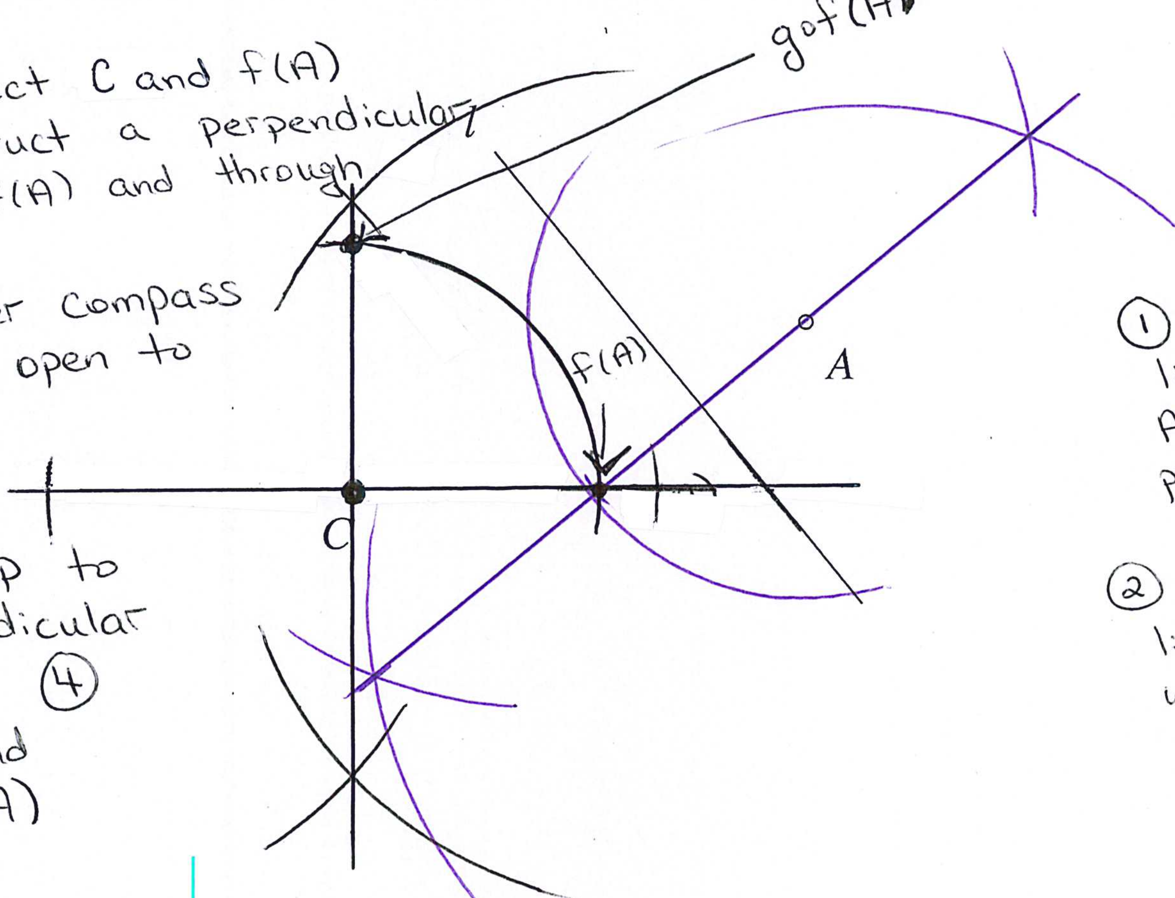
Thus,

$$17711 = f_{21} + 6765$$

$$\Rightarrow f_{21} = 17711 - 6765 = 10946$$

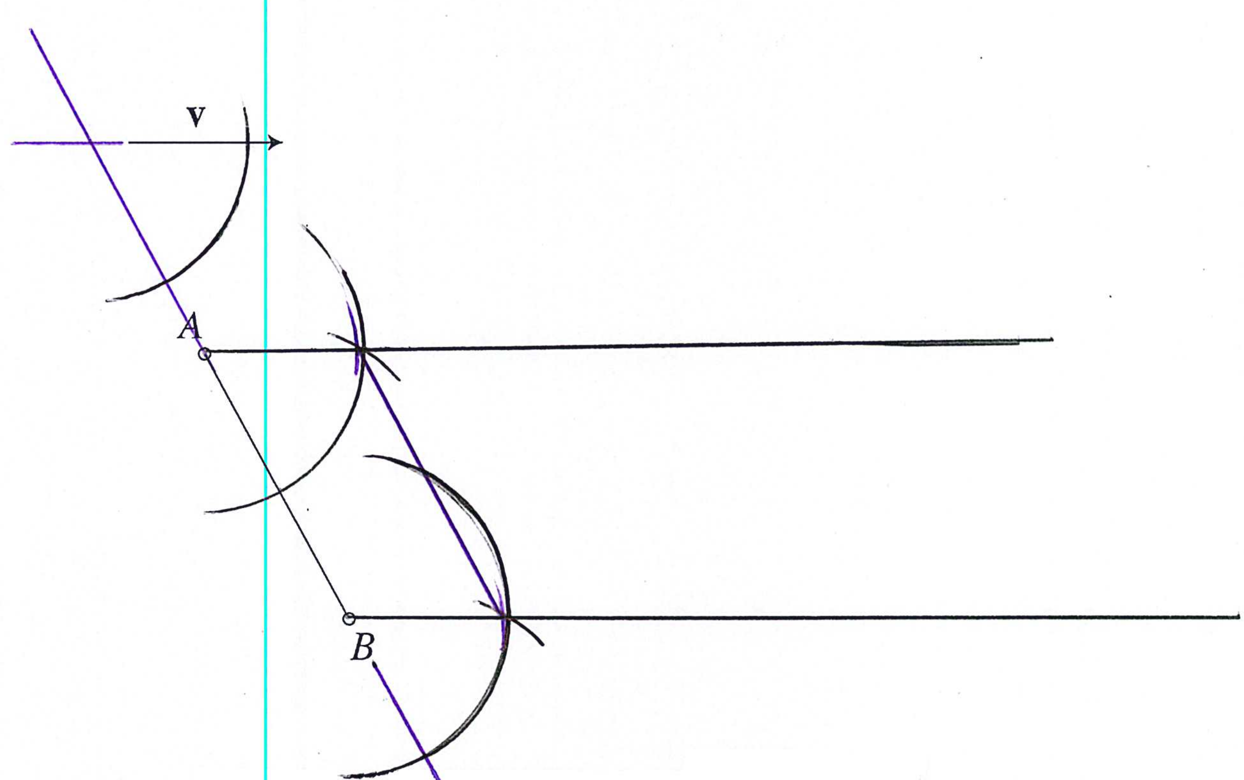
[10 points] 4. [6] (a) Find the image of the point A (shown below) under the composition $g \circ f$ of f followed by g, where f is the reflection with respect to the line l (shown below), and g is the rotation around the center C (shown below) through an angle of 90°.

③ Connect C and f(A)
④ Construct a perpendicular to Cf(A) and through C
⑤ Center compass at C, open to f(A) and arc up to perpendicular from ④ to find gof(A)



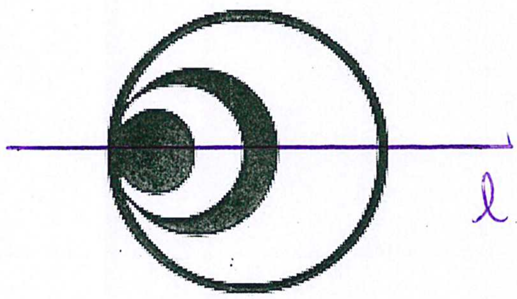
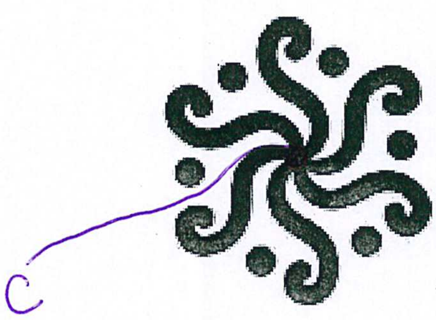
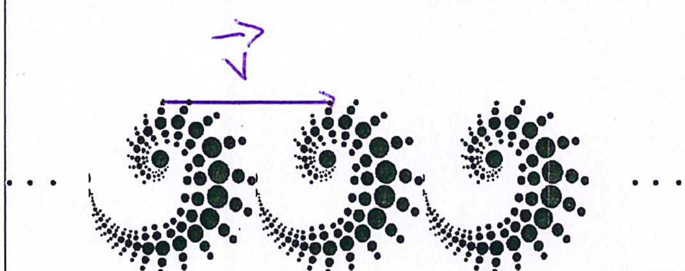
① Find line through A and perpendicular to l
② f(A) is on line from ① with same distance from l as A

(b) [4] Construct the image of the line segment AB (shown below) under the translation with respect to the vector v (shown below).



① Construct a line through A and parallel to $\vec{v}$
② Find length of $\vec{v}$ and move A along parallel line from ① the distance of $\vec{v}$
③ Repeat ① and ② for B.
④ Connect images of A and B.

[13 points] 5. Find the group of symmetries of each of the three objects shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation. If there are reflections, show the line of reflection. If you use translations, describe the vectors of the translation, drawing **precisely** at least one of them. [The objects are defined by the black points.]

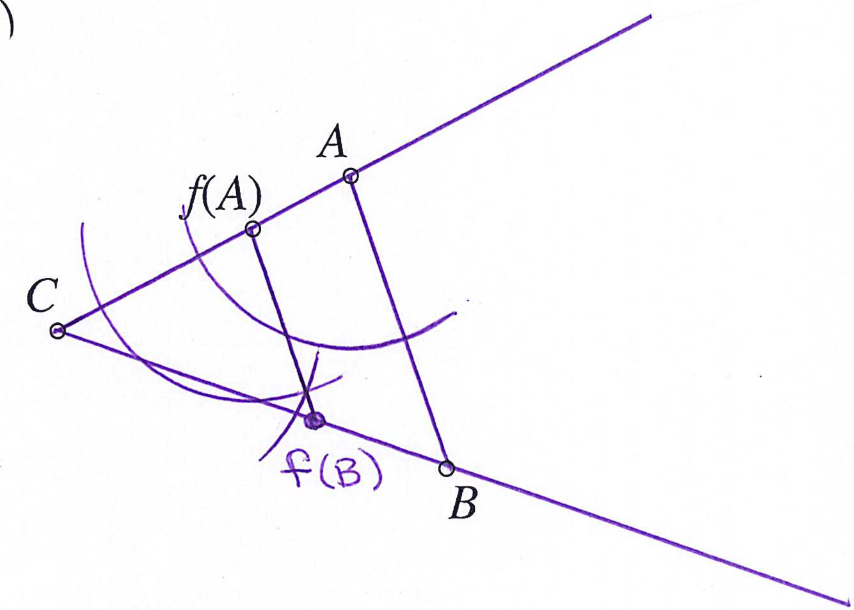
<table>
<tr><th>OBJECT</th><th>THE GROUP OF SYMMETRIES</th></tr>
<tr><td>
</td><td>(a) [2]
$\{$Identity, refl$(\ell)\}$</td></tr>
<tr><td>
</td><td>(b) [5]
$\{$Identity, rot$(C, 60^\circ)$, rot$(C, 120^\circ)$, rot$(C, 180^\circ)$, rot$(C, 240^\circ)$, rot$(C, 300^\circ)\}$</td></tr>
<tr><td>

[This is a Frieze pattern and it extends without end both to the left and to the right.]</td><td>(c) [6]
$\{$Identity, trans$(\vec{v})$, trans$(2\vec{v})$, trans$(3\vec{v})$, ..., trans$(-\vec{v})$, trans$(-2\vec{v})$, trans$(-3\vec{v})$, ...$\}$</td></tr>
</table>

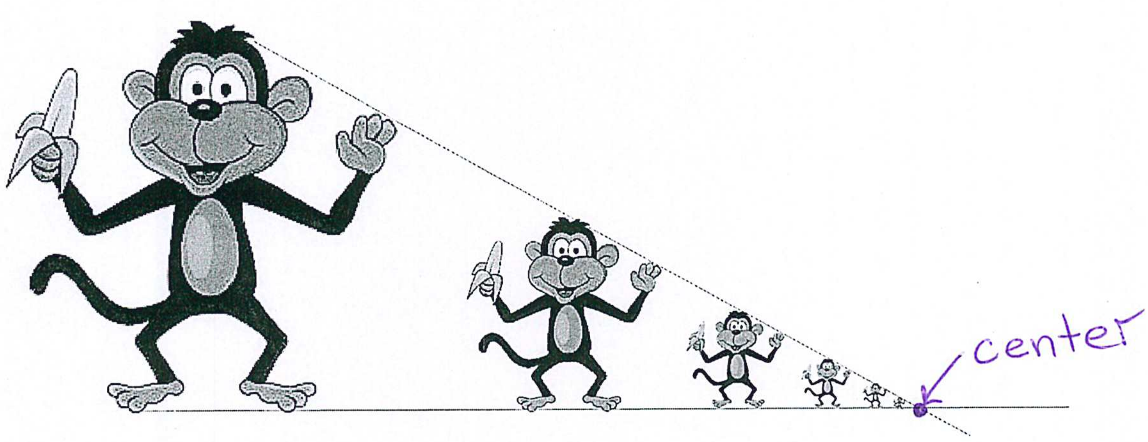
[9 points] 6.

(a) [5] Let f be the central similarity centered at the point C (shown below). We are also given the points A and B, and the point $f(A)$ obtained by applying f to A. Construct the point $f(B)$ obtained by applying f to B.

① Connect C, A, f(A)
② Connect C, B
③ Connect A, B
④ Construct a parallel line to AB through f(A) to get f(B).



(b) [4] In the figure below every funny monkey except the first one is twice smaller than the funny monkey immediately to the left of it. This pattern of monkeys continues without end, and the final outcome is a fractal called F. Confirm that the result is indeed a fractal: find a proper central similarity f that sends F within itself. You get the full marks if you indicate in the figure the center of your central similarity f, and write down the value of the stretching factor α of your central similarity.



$\alpha = \frac{1}{2}$