The Classic Problem

From *Liber Abaci* by Pisano Fibonacci (around AD 1200; also introduced the Hindu-Arabic numeral system to Western Europe):

> A certain man puts a pair of rabbits in a place surrounded on all sides by a wall. How many pairs of rabbits can be produced from that pair in a year if it is supposed that every month each pair begets a new pair, which from the second month on become productive?

- Implicitly assuming that no rabbit dies!

The Rabbit Problem: The First Few Months

Month	1	2	3	4	5	6	7	8
Total Pairs	1	1	2					

Some Notation and Observations

- Let f_n be the number of pairs of rabbits after n months.

We have:

The Recursive Definition

The *Fibonacci numbers* are the numbers in the sequence defined by

$$f_1 = 1$$
$$f_2 = 1$$
$$f_n = f_{n-1} + f_{n-2}$$

Example With Recursive Definition

Given that $f_{19} = 4181$ and $f_{16} = 987$, what are f_{17} and f_{18}?

An Explicit Formula

Binet's Formula for the Fibonacci Numbers:

$$f_n = \frac{(1+\sqrt{5})^n - (1-\sqrt{5})^n}{2^n\sqrt{5}}$$

An Approximation

- Recall: $\varphi = \frac{1+\sqrt{5}}{2} \approx 1.618$ is the Golden Ratio.

Look at what happens as $n \to \infty$:

$$f_n \approx \frac{\varphi^n}{\sqrt{5}}$$

A Cute Conversion

- $\varphi \approx$ the number of kilometers in a mile! (Exact $= 1.609344$.)

E.g.: Convert 30 kilometers to its equivalent in miles:

$$30 = 21 + 8 + 1 = f_8 + f_6 + f_2 \approx \frac{\varphi^8}{\sqrt{5}} + \frac{\varphi^6}{\sqrt{5}} + \frac{\varphi^2}{\sqrt{5}}.$$

Then the number of miles in 30 kilometers would be approximately

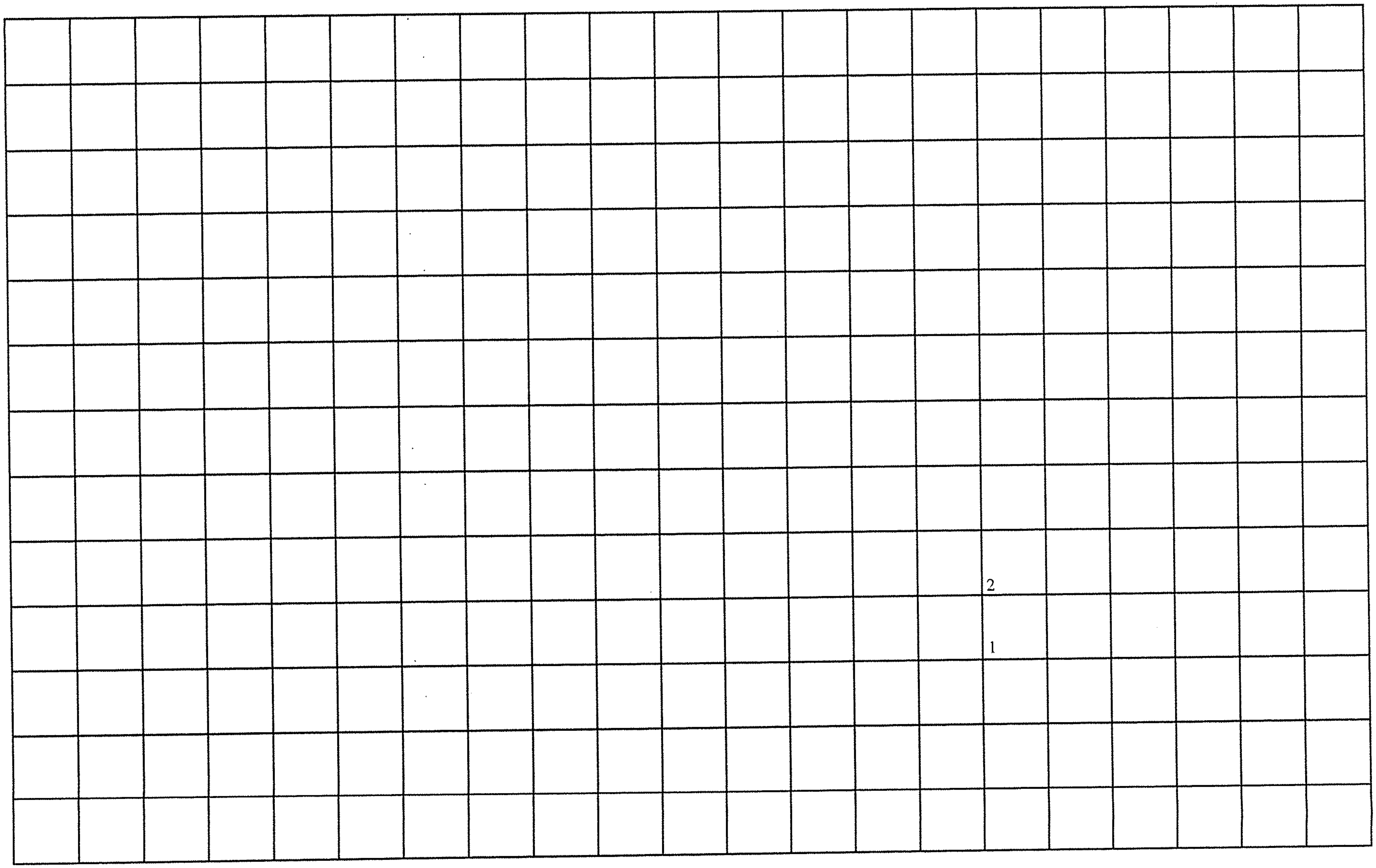
$$\frac{30}{\varphi} \approx \frac{\varphi^7}{\sqrt{5}} + \frac{\varphi^5}{\sqrt{5}} + \frac{\varphi}{\sqrt{5}} \approx f_7 + f_5 + f_1 = 13 + 5 + 1 = 19.$$

The actual number of miles in 30 kilometers is 18.64.

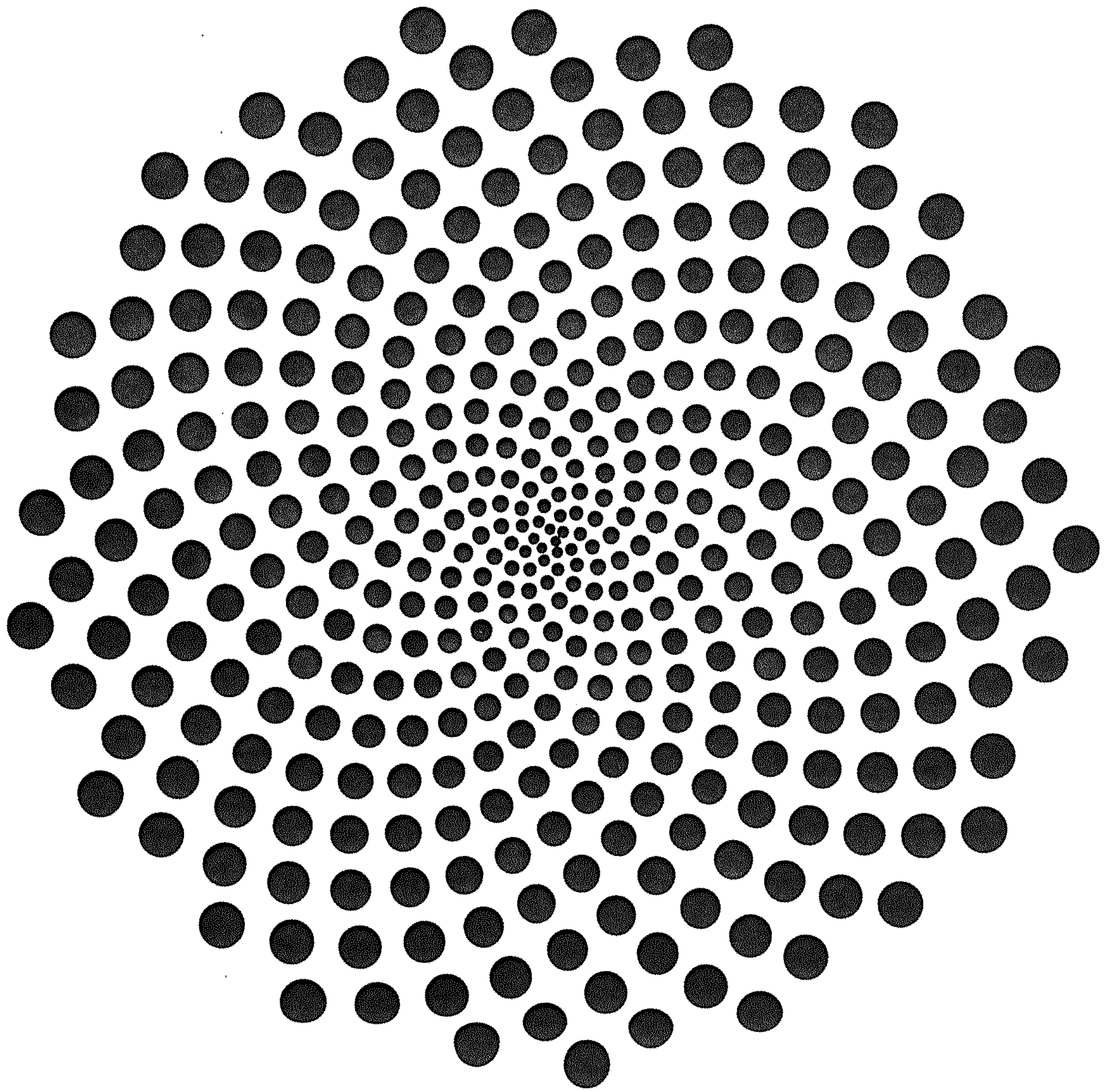
Ratios of Consecutive Fibonacci Numbers

Ratio of Fibonacci Numbers	Ratio	Decimal Equivalent
f_2/f_1	1/1	
f_3/f_2	2/1	
f_4/f_3	3/2	
f_5/f_4	5/3	
f_6/f_5	8/5	
f_7/f_6	13/8	
f_8/f_7	21/13	
f_9/f_8	34/21	
f_{10}/f_9	55/34	
f_{11}/f_{10}	89/55	

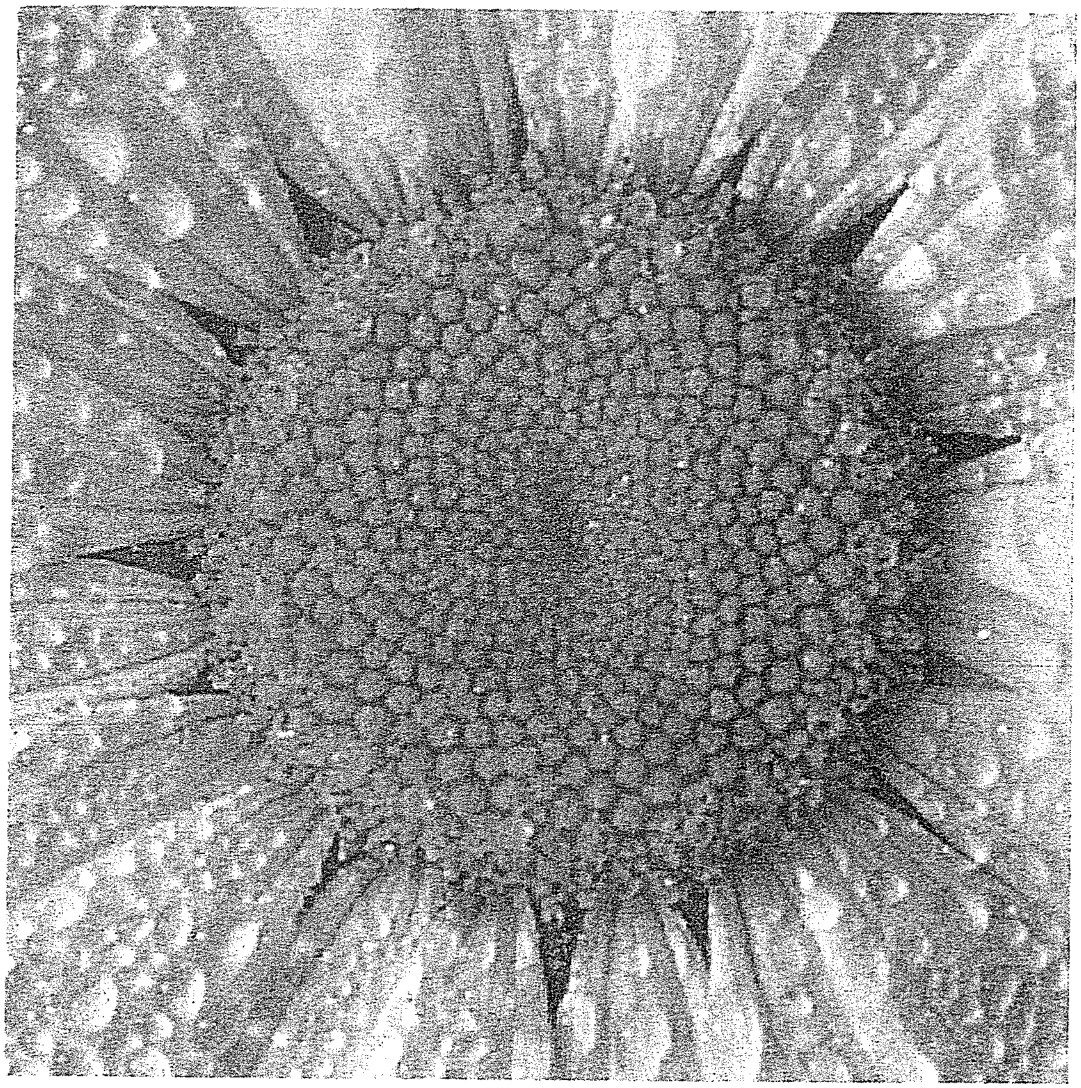
Fibonacci Spiral



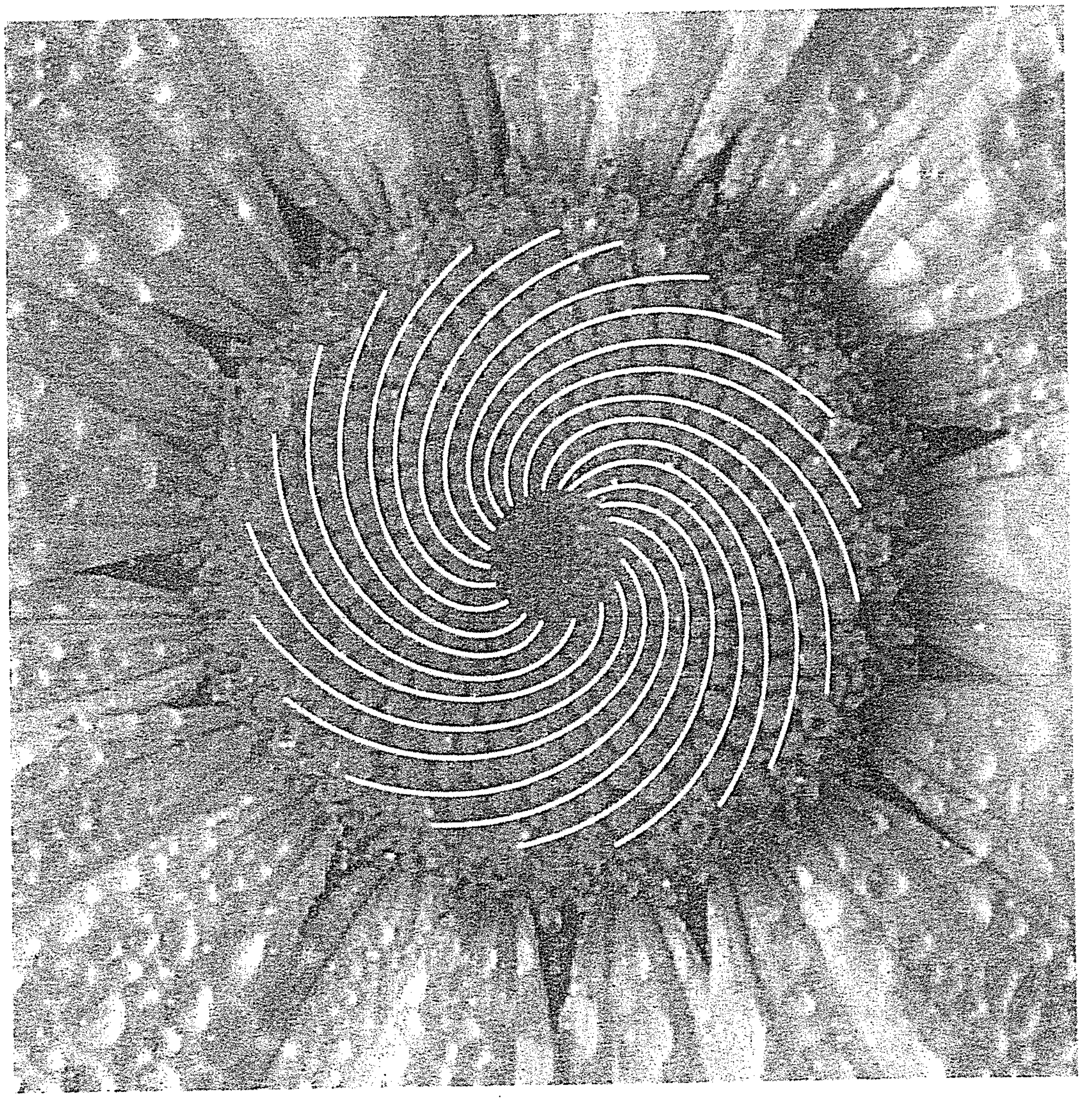
Fibonacci Flowers



Template 2.2.1a Daisy

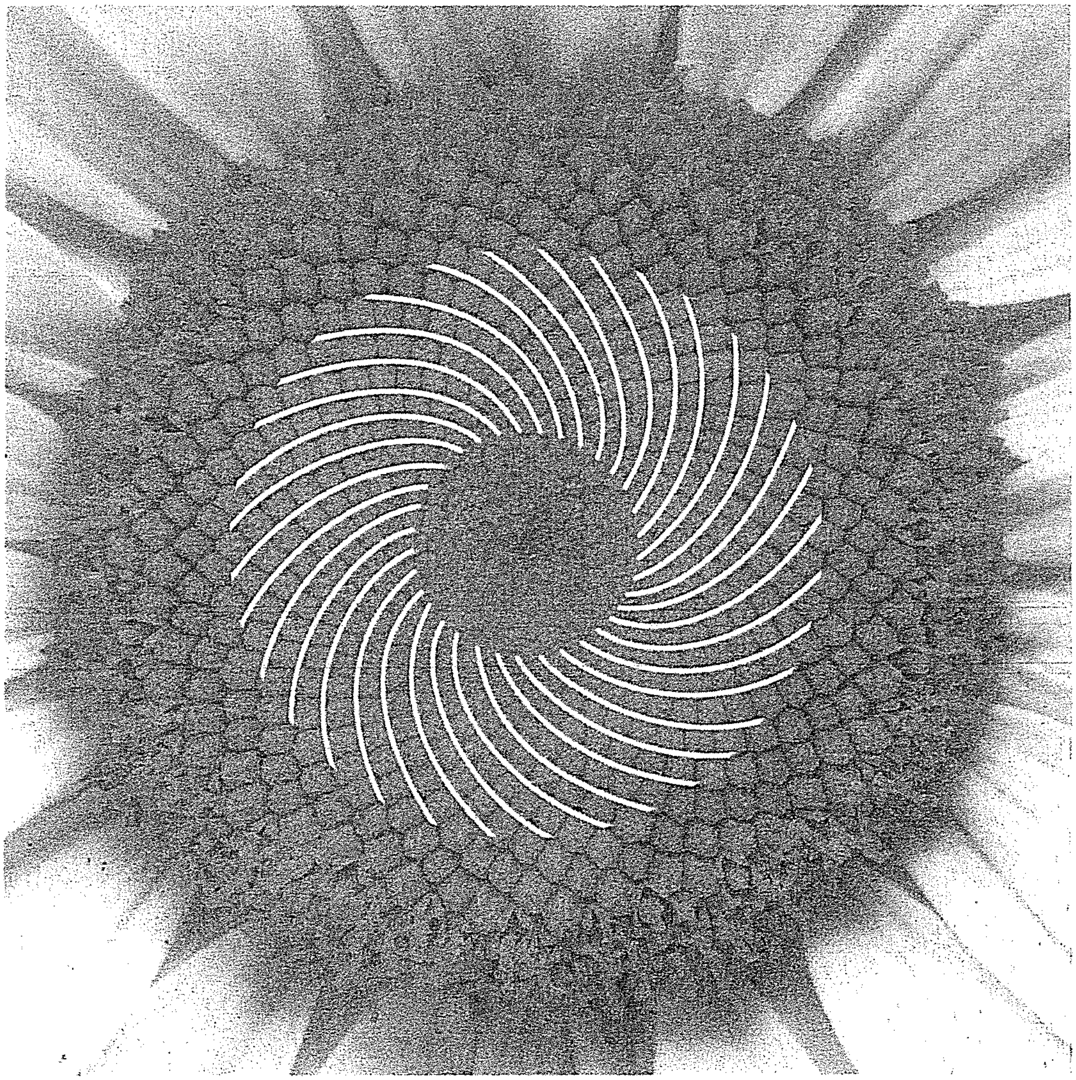


Template 2.2.1b Overlay of daisy with spirals marked



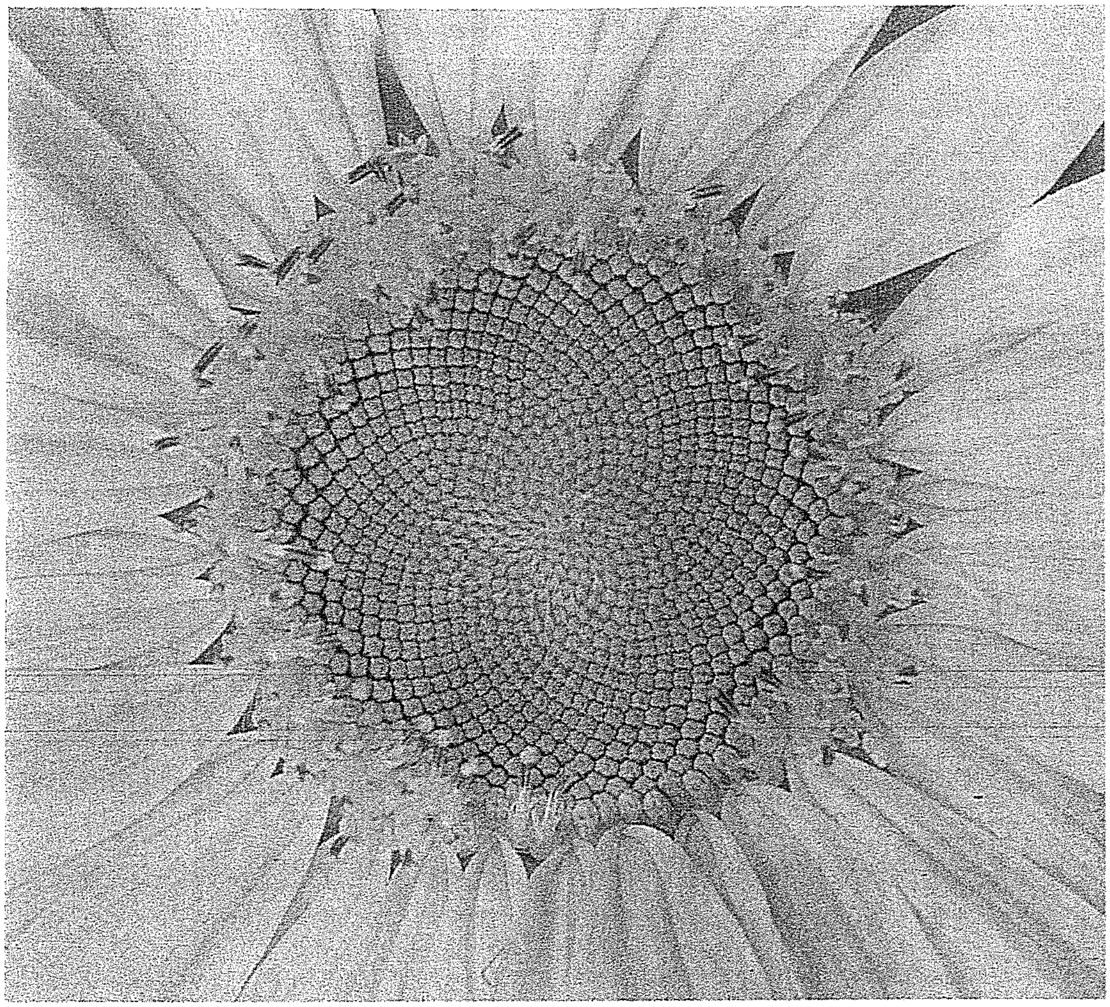
21 spirals

Template 2.2.1c Overlay of daisy with reverse spirals marked



34 spirals

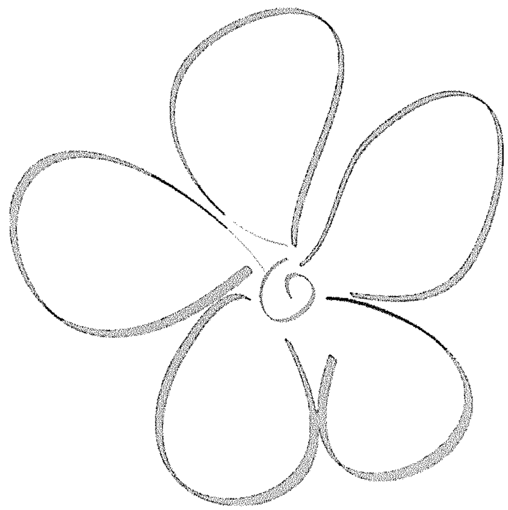
Template 2.2.2 Sunflower spirals



More Fibonacci Numbers And Nature

A great reference to check out is:

http://www.maths.surrey.ac.uk/hosted-sites/R.Knott/Fibonacci/fibnat.html#section3



QUESTIONS???