UNIVERSITY OF MANITOBA
DEPARTMENT OF MATHEMATICS
MATH/FA 1020 Math in Art
FINAL EXAM; April 25, 2017

LAST NAME: (Print in ink) ____________________________________

FIRST NAME: (Print in ink) ____________________________________

STUDENT NUMBER: (in ink) ____________________________________

SIGNATURE: (in ink) ____________________________________
(I understand that cheating is a serious offense)

- ☐ A01 Michelle Davidson
- ☐ A02 Sasho Kalajdzievski

INSTRUCTIONS TO STUDENTS:

Fill in all the information above.

No texts, notes, cell phones or other aids are permitted.

Simple calculators are permitted, as is a ruler and a compass.

Show your work clearly for full marks.

This exam has a title page, 8 pages of questions and 1 blank page at the end for rough work.
Please check that you have all pages.

The value of each question is indicated in the left-hand margin beside the statement of the question. The total value of all questions is 70.

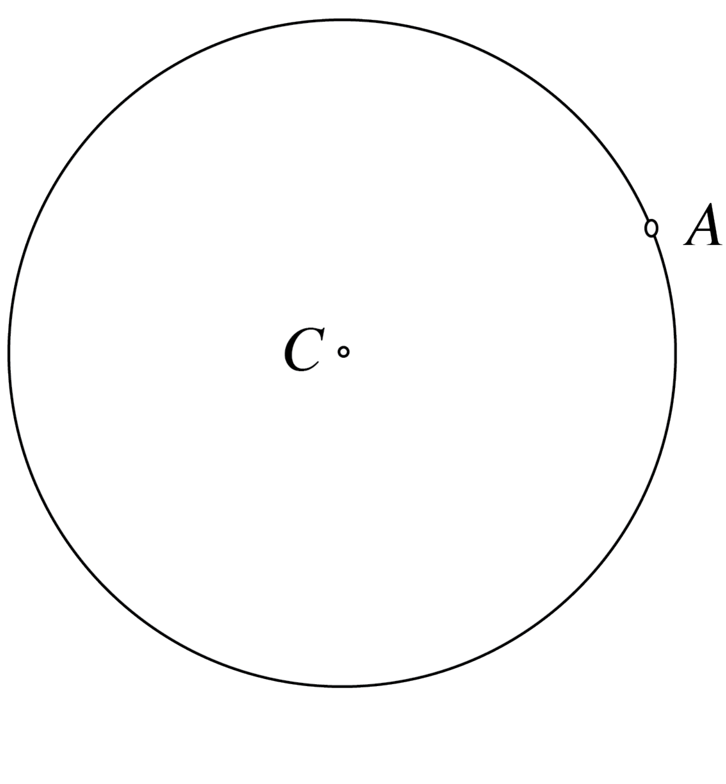
Answer all questions on the exam paper in the space provided. If you need more room, you may continue your work on the reverse side of the page, but clearly indicate that your work is continued there.

Question	**Points**	**Score**
1	6	
2	8	
3	8	
4	8	
5	8	
6	8	
7	9	
8	9	
Total:	64	

Important: **The term "construct" in all of the questions means "construct using an unmarked ruler and a compass". The phrase "unmarked ruler" stands for any ruler that may be used only as a straight edge to draw straight line segments. When you use a compass, show the (intermediate) circular arcs you draw in your constructions (do not erase them). Use words to describe BRIEFLY what you have done.**

[6] 1. [3] (a) Construct a line that intersect the given line at 45°.

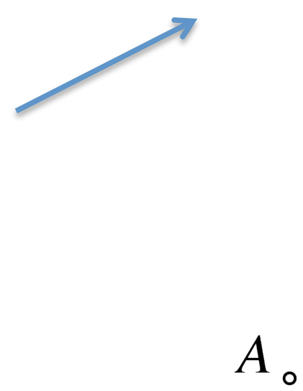
[3] (b) Construct a circle that passes through the point *A* and is perpendicular to the given circle (with center *C*). Clearly indicate the center of the circle you have constructed.



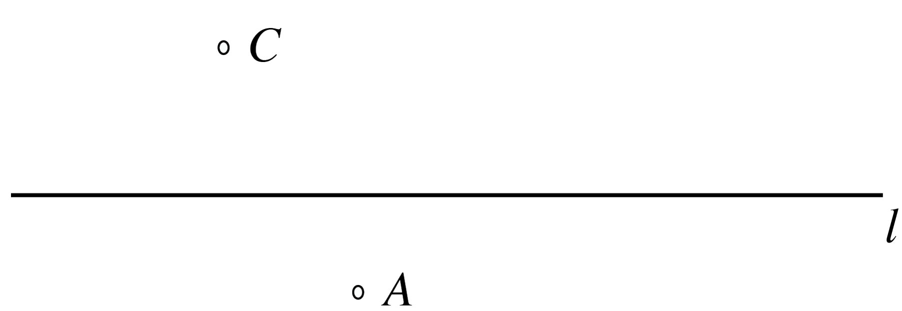
[8] 2. [3] (a) Construct a regular hexagon so that the line-segment shown below is one of its sides.

[5] (b) Construct a regular pentagon so that the line-segment shown below is one of its sides.

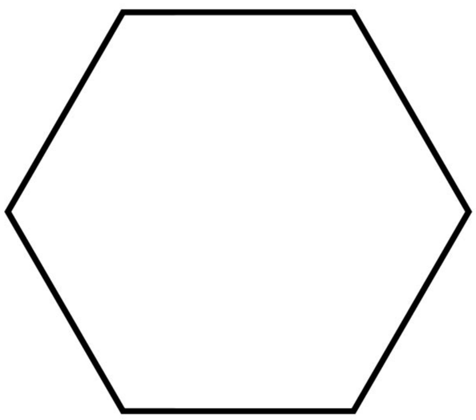
[8] 3. [2] (a) The vector **v** and the point A are shown; construct the image of A under the translation along the vector **v.**



[6] (b) The line l and the points A and C are as given. Denote for shortness $f = rot(C, 60^{\circ})$ (rotation around C through 60º), and $g = ref_l$ (reflection with respect to the line l). Construct the image $g\big(f(A)\big)$ of the point A under the composition of f followed by g.

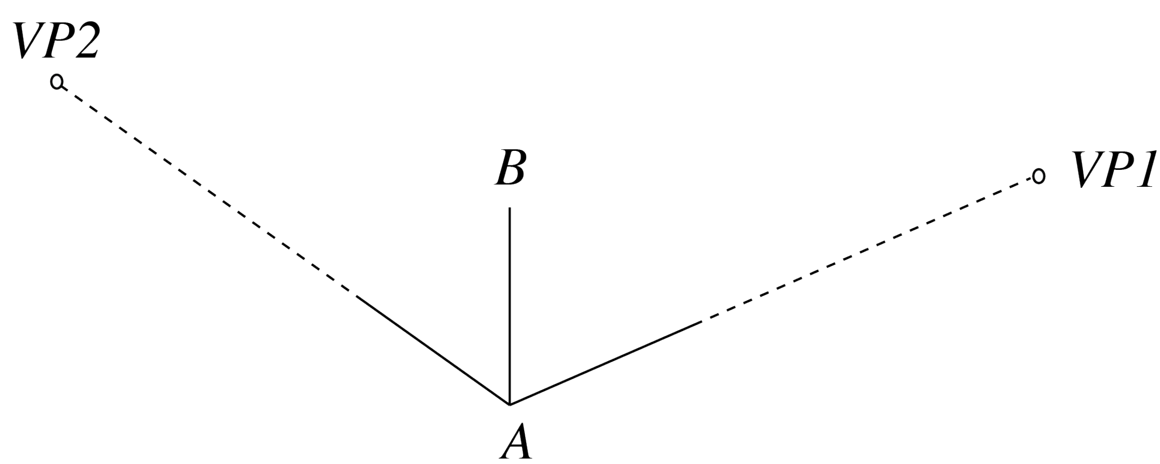


[8] 4. [5] (a) Find the group of the regular hexagon shown below. If you claim a rotational symmetry, indicate the center of the rotation and the angle of rotation (in degrees). If there are reflections, show (and denote) the line of reflection.

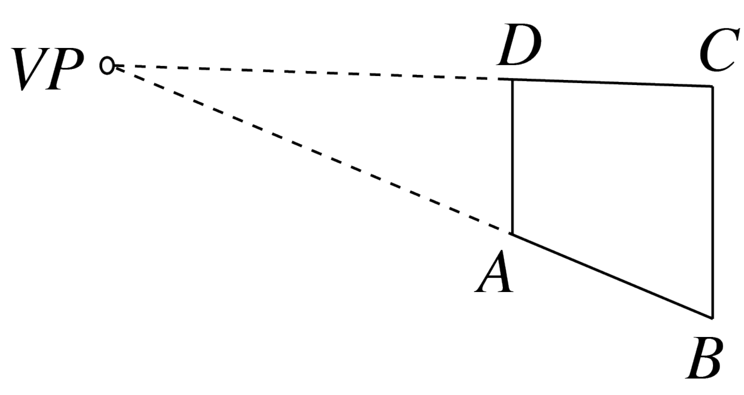


[3] (b) Sketch any object in the plane that has exactly 5 symmetries. Write the group of 5 symmetries of the object you sketch.

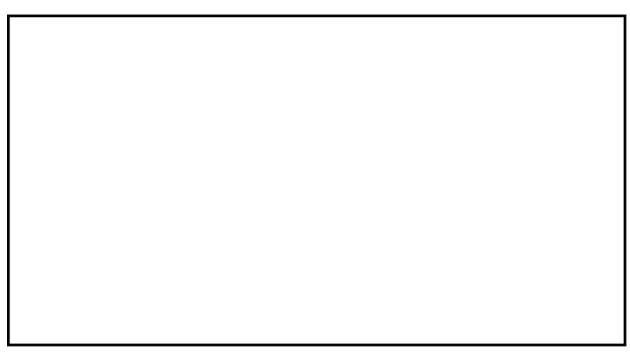
[8] 5. [5] (a) Three adjacent edges of a box are shown in 2-point perspective, with vanishing points VP1 and VP2 (the edge AB is vertical and parallel to the drawing plane). Construct the rest of the (edges of the) box. (Note: the dashed lines are not a part of the box.)



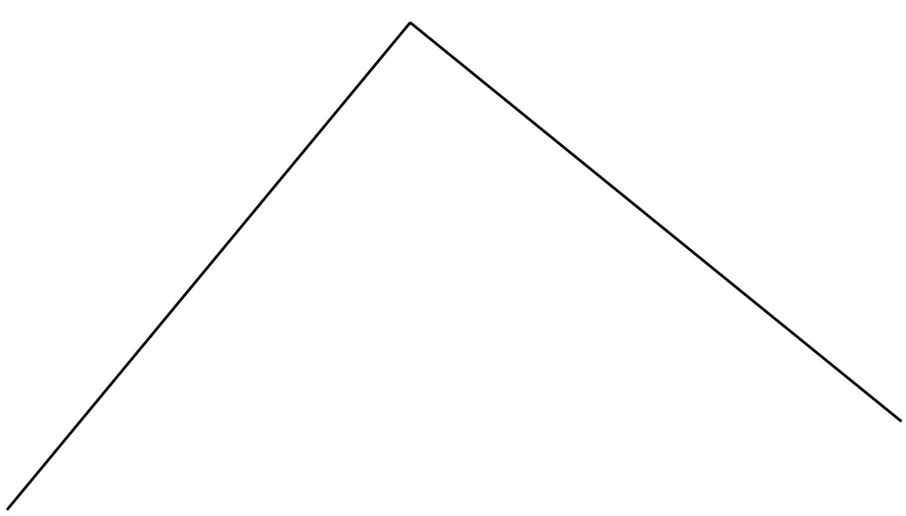
[3] (b) The rectangle ABCD is shown in perspective, where VP is a vanishing point and the edges BC and AD are vertical and parallel to the drawing plane. Construct the subdivision of the rectangle ABCD into 4 equal smaller rectangles.



[8] 6. [**5**] **(a)** Construct at least four (tangent) lines outlining the contour of the top of the ellipse inscribed in the given rectangle. (**No** need to construct the bottom part of the ellipse, so that the constructed one half of an ellipse should be in the top half of the given rectangle.)



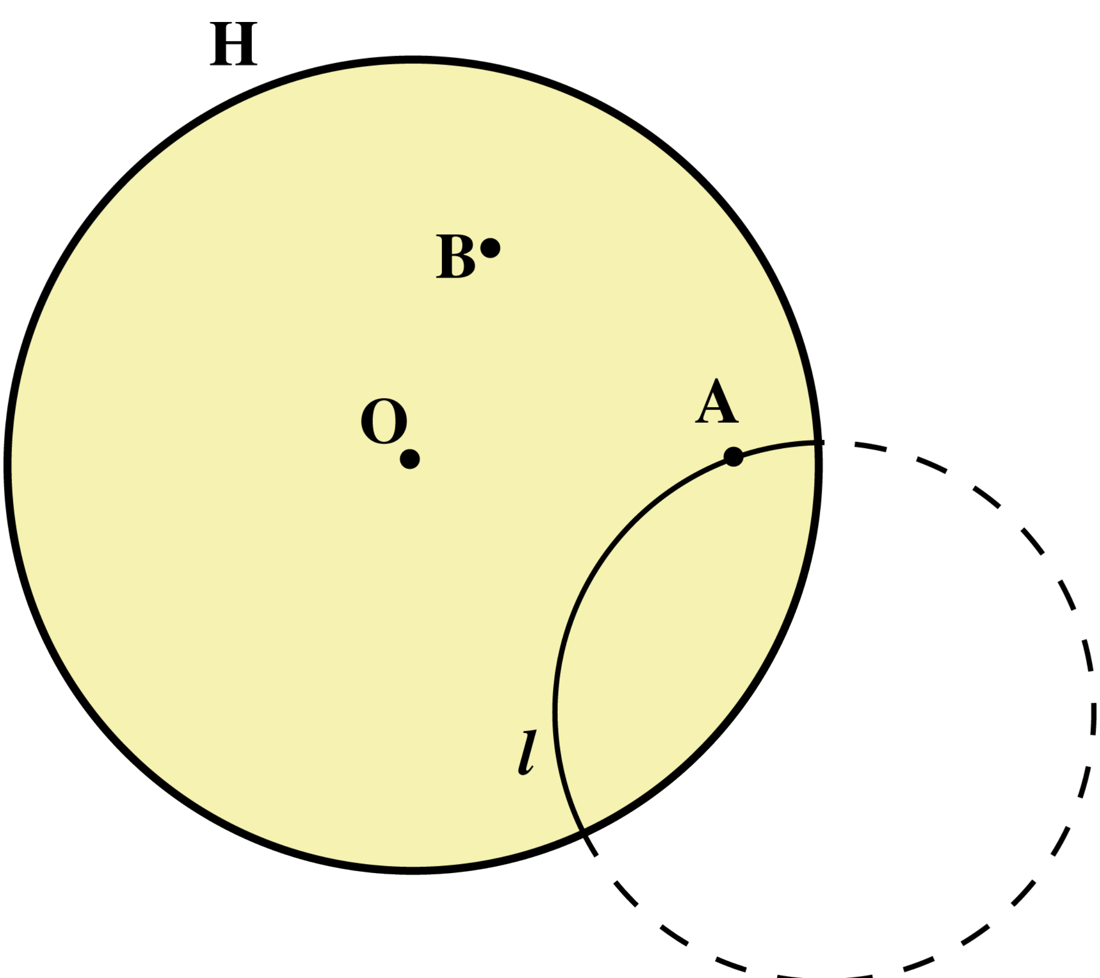
[3] (b) Subdivide each of the two segments shown below into 4 equal parts, then use these points to construct an outline of a parabola by drawing precisely 4 tangential line segments.



[9] 7. We are given a hyperbolic line l, the point A on that hyperbolic line, and a point B outside the line l.

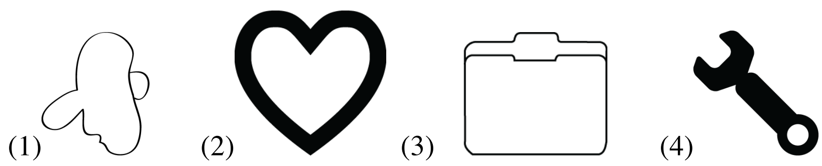
[2] (a) Construct one hyperbolic line parallel to l and passing through B. Label that line by m.

[7] (b) Construct the hyperbolic line passing through A and perpendicular to l; label it by n.

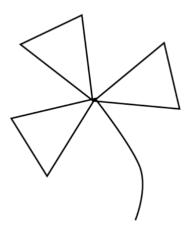


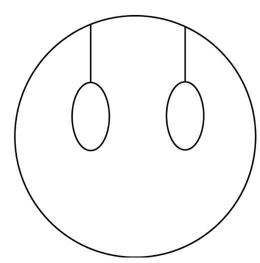
[9] 8. The objects depicted in parts (a) and (b) of this question consist of the black-coloured points only.

[2] (a) Which of the following four designs are mutually homotopic? (No justification required.)



[3] (b) Show that the two designs shown below are homotopic by drawing at least three in-between sketches showing the one of the objects can be continuously deformed into the other.





[4] (c) Consider the **surface** of the double-donut (connect sum of two tori).
i) Find the genus of this surface. Justify by drawing the associated circular cuts.
ii) Find the Euler characteristic of this surface. Justify by giving the appropriate formula used.