Problem Set 9
Due: Wednesday, April 1

For this assignment, let $n_p(G)$ denote the number of Sylow p-subgroups of a group G.

1. Let G be a group of order 385. Prove that the center of G contains a Sylow 7-subgroup of G.
2. Prove that a group of order 132 is not simple.
3. Let A and B be finite groups and let p be a prime.
   (a) Prove that any Sylow p-subgroup of $A \times B$ is of the form $P \times Q$, where P is a Sylow p-subgroup of A and Q is a Sylow p-subgroup of B.
   (b) Prove that $n_p(A \times B) = n_p(A) \cdot n_p(B)$.
4. Suppose that the degree of the field extension $F \subseteq K$ is a prime p. Show that any subfield E of K containing F is either K or F.