Problem Set 13
Due: Wednesday, May 6

1. A field K of characteristic $p > 0$ is called *perfect* if every element of K is a p^{th} power in K. Suppose K is a field of characteristic $p > 0$ which is not a perfect field. Show that there exist irreducible inseparable polynomials over K and conclude that there exist inseparable finite field extensions of K.

2. Let K be a splitting field for $f(x) = (x^2 - 2)(x^2 - 3)(x^2 - 5)$ over $\mathbb{Q}$.

   (a) Prove that the Galois group of the extension $\mathbb{Q} \subseteq K$ is isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$.

   (b) Determine all the intermediate fields $\mathbb{Q} \subseteq F \subseteq K$.

3. Let K be a splitting field for $f(x) = x^4 - 14x^2 + 9$ over $\mathbb{Q}$. Show that the Galois group of the extension $\mathbb{Q} \subseteq K$ is isomorphic to the Klein 4-group (i.e., $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}/2\mathbb{Z}$).