# Problem Set 11
## Due: Wednesday, April 22

1. Suppose $F = \mathbb{Q}(\alpha_1, \alpha_2, \ldots, \alpha_n)$ where $\alpha_i^2 \in \mathbb{Q}$ for $i = 1, 2, \ldots, n$. Prove that $\sqrt[3]{2} \notin F$.

2. Using the following steps, determine the degree of $\beta := 1 + \sqrt[3]{2} + \sqrt[3]{4}$ over $\mathbb{Q}$.
   (a) Let $\alpha = \sqrt[3]{2}$. Show that $\beta \in \mathbb{Q}(\alpha)$.
   (b) Find $[\mathbb{Q}(\alpha) : \mathbb{Q}]$.
   (c) Explain why $[\mathbb{Q}(\beta) : \mathbb{Q}]$ divides $[\mathbb{Q}(\alpha) : \mathbb{Q}]$.
   (d) Show that $[\mathbb{Q}(\beta) : \mathbb{Q}] \neq 1$.
   (e) What is the degree of $\beta$ over $\mathbb{Q}$?

3. Let $\varphi : K \to L$ be a field extension. Assume that $L$ is algebraically closed, and let $F = \{u \in L \mid u \text{ is algebraic over } K\}$. Show that $F$ is an algebraic closure of $K$.

4. Let $K$ be a finite field. Show that $K$ is not algebraically closed. (*Hint:* Let $K = \{a_1, \ldots, a_n\}$ with $a_1 \neq 0$ and consider $f = a_1 + (x - a_1)(x - a_2) \cdots (x - a_n) \in K[x]$.)