MATH 721, Algebra II
Exercises 2
Due Wed 28 Jan

Throughout this homework set, let R be a commutative ring with identity.

Exercise 1. Let M and N be R-modules. Prove that there is an R-module isomorphism $M \otimes_R N \cong N \otimes_R M$.

Exercise 2. Let M be an R-module, and let I be an ideal of R. Prove that there is an R-module isomorphism $(R/I) \otimes_R M \cong M/IM$.

Exercise 3. Let M be an R-module. Prove that the following conditions are equivalent.

(i) M is flat over R.

(ii) For every R-module monomorphism $g' \colon N' \to N$, the induced homomorphism $M \otimes_R g' \colon M \otimes_R N' \to M \otimes_R N$ is a monomorphism.

(iii) For every short exact sequence $0 \to N' \xrightarrow{g'} N \xrightarrow{g} N'' \to 0$ of R-module homomorphisms, the induced sequence

$$0 \to M \otimes_R N' \xrightarrow{M \otimes_R g'} M \otimes_R N \xrightarrow{M \otimes_R g} M \otimes_R N'' \to 0$$

is exact.