Problem Set 2

Due: 12:00 p.m. on Wednesday, October 5

Instructions: Work all of the following problems. A subset of the problems will be graded. Your grade will also reflect your participation in solution presentations. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets*. Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: For this Problem Set, assume that all rings are non-zero and contain an identity. We also let k be a field and $x_0, \ldots, x_n$ be indeterminates.

1. Find all possible Hilbert functions for 9 distinct points in $\mathbb{P}^2$. Pick one of the Hilbert functions $\mathcal{H}$ and find a set $\mathbb{X} \subset \mathbb{P}^2$ of 9 distinct points in $\mathbb{P}^2$ such that $H(\mathbb{X}) = \mathcal{H}$. How do you know that the constructed set of points has the selected Hilbert function?

2. For parts (b) - (d) of this exercise use graded reverse-lexicographic order with $x_1 >_{grevlex} > x_2 >_{grevlex} \cdots$.

   (a) Find a $(3,4,5)$-lex-plus-powers ideal $L \subset S = k[x_1, x_2, x_3]$ such that $H(S/L, 3) = 9$ and $H(S/L, 6) = 5$.

   (b) Fix m to be a monomial of degree d in $S = k[x_1, x_2, x_3, x_4]/(x_1^5, x_2^4, x_3^4, x_4^3)$. Recall that $L(m)$ denotes the set of all degree d monomials in S which are greater than or equal to m. Decompose $|L(x_1^3 x_2^3 x_4^2)|$ in terms of integers of the form $\binom{e_1,\ldots,e_j}{l}$. Give an algebraic description of each term in the decomposition.

   (c) Assume $I \subset S = k[x_1, x_2, x_3, x_4]$ is a homogeneous ideal containing $\{x_1^5, x_2^4, x_3^4, x_4^3\}$. If $H(S/I, 8) = 17$, then what is the largest value possible for $H(S/I, 9)$?

   (d) Assume that the EGH Conjecture is true. Can there be a homogeneous $(3,4,4,5)$-ideal $I \subset S = k[x_1, x_2, x_3, x_4]$ with $H(S/I) = (1, 4, 10, 18, 24, 29, \ldots)$?

3. A homogeneous ideal $I \subset R = k[x_1, x_2, x_3]$ has the following Betti diagram:

total	1	5	6	2
0	1	-	-	-
1	-	2	-	-
2	-	2	4	-
3	-	-	-	1
4	-	1	2	1

   Find the minimal graded free resolution of R/I.