Problem Set 14

Due: 3:00 p.m. on Wednesday, December 2

Instructions: Carefully read Section 12.1 of the textbook. Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined on the sheet *Guidelines for Problem Sets.* Submit your solutions in-class or to Dr. Cooper's mailbox in the Department of Mathematics.

Exercises: For this Problem Set, let R and S be commutative rings with identity and let M be a unital R-module.

1. Prove that the following conditions are equivalent:
   (i) M is a Noetherian R-module;
   (ii) Every submodule $N \subseteq M$ is a Noetherian R-module;
   (iii) For every submodule $N \subseteq M$, the quotient M/N is a Noetherian R-module.
2. Let $\alpha : S \to R$ be a surjective ring homomorphism. Prove that M is a Noetherian S-module if and only if M is a Noetherian R-module.
3. Let R be an integral domain and let M be a projective R-module.
   (a) Prove that M is torsion free.
   (b) Assume further that R is a P.I.D. and that M is finitely generated. Prove that M is free.

Bonus. Let R be a Noetherian ring and $\alpha : R \to R$ be a surjective ring homomorphism. Prove that α is an isomorphism.