Lab 8

Extreme Values, The Mean Value Theorem, Monotonicity, and the Shape of a Graph

1. Find the critical points of the following functions.

   (a) $f(x) = x^3 - 4x^2 + 4x$

   (b) $f(x) = x^2(x+2)^3$

   (c) $g(\theta) = \sin^2(\theta) + \theta$

2. Find the extreme values on the interval.

   (a) $f(x) = x(10 - x)$, $[-1, 3]$

   (b) $g(\theta) = \sin^2\theta - \cos\theta$, $[0, 2\pi]$

   (c) $f(x) = x - 12\ln x$, $[5, 40]$

3. Verify that $f(x) = x\sqrt{x+6}$ satisfies the hypotheses of Rolle's Theorem on the interval $[-6, 0]$. Then find all numbers c that satisfy the conclusion of Rolle's Theorem.

4. Verify that $f(x) = f(x) = x^3 + x - 1$ satisfies the hypotheses of the Mean Value Theorem on the interval $[0, 2]$. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

5. Show that the equation $2x - 1 - \sin x = 0$ has exactly one real root.

6. Suppose that $f'(x) \leq 2$ for $x > 0$ and $f(0) = 4$. Show that $f(x) \leq 2x + 4$ for all $x \geq 0$.

7. Use the First Derivative Test to determine whether the critical point(s) is a local max or min (or neither).

   (a) $y = \dfrac{1}{x^2+1}$

   (b) $f(x) = \cos^2 x + \sin x$ on $(0, \pi)$

   (c) $f(x) = \dfrac{1}{3}x^3 - x^2 + x$

8. Find the points of inflection for the following functions.

   (a) $y = x^3 - 4x^2 + 4x$

   (b) $f(x) = (x^2 - x)e^{-x}$

9. Determine the intervals on which f is concave up or concave down.

   (a) $f(x) = 3x^5 - 5x^4 + 1$

   (b) $f(x) = x^{5/3}$

   (c) $f(x) = \dfrac{3x}{x^2-4}$