Lab 5

The Derivative: Definition, Product & Quotient Rules, and Rates of Change

1. *Use the limit definition* to calculate $\frac{dy}{dx}$ for the following functions.
   (a) $y = 4 - x^2$
   (b) $y = \frac{1}{2-x}$
   (c) $y = \sqrt{2x+1}$

2. Each limit represents a derivative $f'(a)$. Find $f(x)$ and a.
   (a) $\lim_{h \to 0} \frac{\sqrt{1+h}-1}{h}$
   (b) $\lim_{x \to \pi} \frac{\sin x \cos x}{x - \pi}$

3. Find the points on the graph of $f(x) = x^3 - 3x^2 + x + 4$ where the tangent line has slope 10.

4. Determine the points c (if any) such that $f'(c)$ does not exist where $f(x) = |x-5|$.

5. Compute the derivative for each of the following functions.
   (a) $y = t^{-7.3}$
   (b) $y = 3x^5 - 7x^2 + 4$
   (c) $y = \frac{x+1}{x^2+1}$
   (d) $y = te^{t-4}$

6. Use the following table of values to calculate the derivative of the given function at $x = 2$.

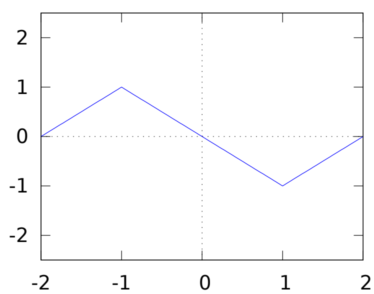
x	$f(x)$	$g(x)$	$f'(x)$	$g'(x)$
2	5	4	-3	9

   (a) $y = 3f(x) - 2g(x)$
   (b) $y = f(x)g(x)$
   (c) $y = \frac{f(x)}{g(x)}$

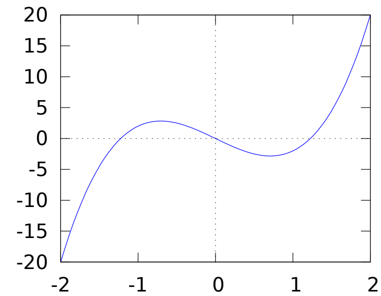
7. A stone is shot with a slingshot vertically upward with an initial velocity of 50 m/s from an initial height of 10 m.
   (a) Find the velocity at $t = 2$ and at $t = 7$. Explain the change in sign.
   (b) What is the stone's maximum height and when does it reach that height?

8. Match the graph of each function in (1) – (4) with the graph of its derivative in (A) – (D). Record your answers in the table below.

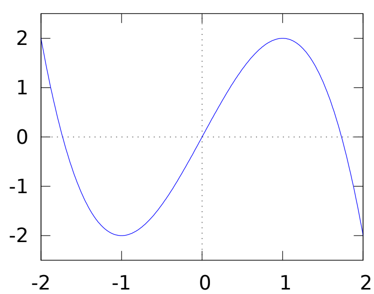
(1)	________	(2)	________
(3)	________	(4)	________



(1)



(A)



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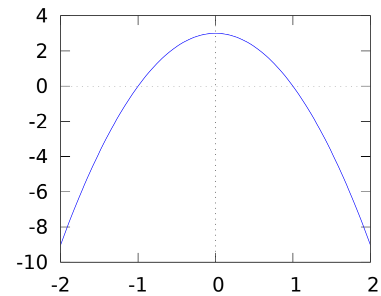
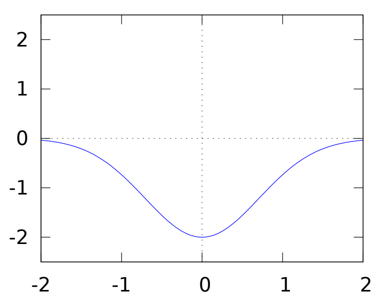


(B)



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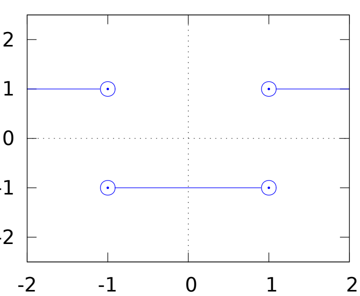
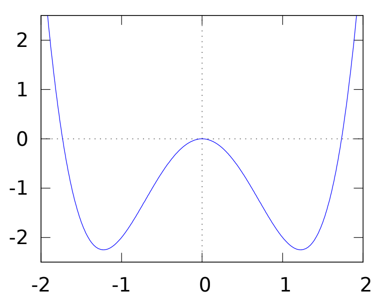
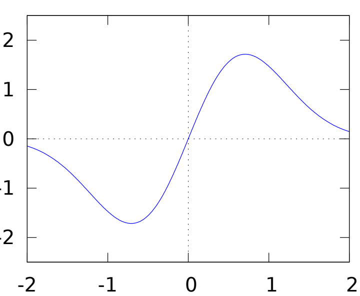


(C)



(4)



(D)