Problem Set 4
Due: Thursday, February 9

Work all of the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined in the *General Problem Set Guidelines Sheet*.

Unless otherwise stated, all problems can be found in the appropriate *Exercises* sections of the text (*Abstract Algebra* by D. Dummit and R. Foote, 3rd Edition).

- Let $\phi : G \to H$ be an isomorphism of the groups G and H. Show that G is cyclic if and only if H is cyclic.
- Section 2.3 # 3, 10, 12 part (b), 16, 26
  - Note for # 12 part (b): You may assume that $Z_2 \times \mathbb{Z}$ is isomorphic to $\mathbb{Z}/2\mathbb{Z} \times \mathbb{Z}$.
  - Note for # 26: $Aut(Z_n)$ is the set of all automorphisms of Z_n. You may assume this is a group under function composition.
- Show that the group $G = (\mathbb{Z}/64\mathbb{Z})^\times$ is not cyclic. (*Hint:* Find two distinct subgroups of G of order 2.)
- Let $H = \langle x \rangle$ be a cyclic group.
  - (i) Let m be an integer. Prove that $\langle x^m \rangle = \langle x^{|m|} \rangle$, where $|m|$ denotes the absolute value of m.
  - (ii) Assume $|H| = \infty$. Prove that for any distinct non-negative integers a and b, $\langle x^a \rangle \neq \langle x^b \rangle$. Deduce that the nontrivial subgroups of H correspond bijectively with the integers $\{1, 2, 3, \ldots\}$.