Quiz Set 3
For Quiz on Thursday, March 6

Work all of the following problems. A subset of the problems and definitions from Chapter 5 will be on Quiz 3 to be given March 6. Quizzes will be graded for correctness and clarity.

(1) (Gallian, Chapter 5 Exercises, #8) Show that A_8 contains an element of order 15.

(2) (Gallian, Chapter 5 Exercises, #10) What is the maximum order of any element in A_{10}?

(3) (Gallian, Chapter 5 Exercises, #15) Let n be a positive integer If n is odd, is an n-cycle an odd or an even permutation? If n is even, is an n-cycle an odd or an even permutation? Be sure to support your answers.

(4) (Gallian, Chapter 5 Exercises, #16) If α is even, prove that α^{-1} is even. If α is odd, prove that α^{-1} is odd.

(5) (Gallian, Chapter 5 Exercises, #17) Prove Theorem 5.6. That is, prove that the set of even permutations in S_n forms a subgroup of S_n.

(6) (Gallian, Chapter 5 Exercises, #25) Give two reasons why the set of odd permutations in S_n is not a subgroup.

(7) (Gallian, Chapter 5 Exercises, #26) Let α and β belong to S_n. Prove that $\alpha^{-1}\beta^{-1}\alpha\beta$ is an even permutation.

(8) (Gallian, Chapter 5 Exercises, #30) Prove that (1 2 3 4) is not the product of 3-cycles.

(9) If G is an Abelian group and n is a fixed positive integer, then $G^n := \{g^n \mid g \in G\}$ is a subgroup of G. (You can assume this fact.)

   (a) Prove that if $G = S_n$, then $G^2 \subseteq A_n$.

   (b) Let $G = A_4$. Prove that G^2 is *not* a subgroup of G.

(10) Prove that the notion of group isomorphisms is transitive. That is, prove that if G, H, and K are groups such that $G \approx H$ and $H \approx K$, then $G \approx K$.

(11) (Gallian, Chapter 6 Exercises, #28) The group $\left\{ \begin{bmatrix} 1 & a \\ 0 & 1 \end{bmatrix} : a \in \mathbb{Z} \right\}$ is isomorphic to what familiar group? What if $\mathbb{Z}$ is replaced by $\mathbb{R}$? Support your answers by verifying specific isomorphisms.

(12) Let a and g be fixed elements of a group G with identity element e.

   (a) Prove that if $x \in C(a)$, then $gxg^{-1} \in C(gag^{-1})$.

   (b) Prove that $C(a)$ is isomorphic to $C(gag^{-1})$.