
Problem Set 4

Due: Thursday, October 24

Work the following problems. A subset of the problems will be graded. Be sure to adhere to the expectations outlined in the *Problem Set Guidelines Sheet*.

- Verify that $17296 = 2^4 \cdot 23 \cdot 47$ and $18416 = 2^4 \cdot 1151$ is an amicable pair.
- Section 8 # 5, 8, 12, 13
- Let a and m be positive integers such that $(a, m) \neq 1$.
 - (a) Show that $a^{\phi(m)} \not\equiv 1 \pmod{m}$ with a specific example.
 - (b) Prove that there is no positive integer k such that $a^k \equiv 1 \pmod{m}$.
- Graduate Students: Section 8 # 14

Hint for Section 8 # 13: Write the even perfect number as $2^{k-1}(2^k - 1)$. Consider the case $k = 2$ separately. For $k > 2$, separate the argument into cases that arise from dividing k by 4. You may assume (without proof) the fact that $16^t \equiv 6 \pmod{10}$ for any positive integer t .