Math 206-01: Linear Algebra I **Dr. S. Cooper, Spring 2007**

Differential Equations Homework

As in class, we let $\mathbb{D}^2$ denote the pairs of all differentiable, real-valued functions and $\delta : \mathbb{D}^2 \to \mathbb{D}^2$ be the linear mapping defined by differentiation.

(1) Let V be the subset of all $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ in $\mathbb{D}^2$ for which:

$$x_1'(t) = -x_2(t), \quad x_2'(t) = -x_1(t).$$

Given the initial conditions

$$\mathbf{x}(0) = \begin{bmatrix} x_1(0) \\ x_2(0) \end{bmatrix} = \begin{bmatrix} 0 \\ 2 \end{bmatrix},$$

find the solutions $\mathbf{x}(t)$ to this differential equation.

(2) Let V be the subset of all $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ in $\mathbb{D}^2$ for which:

$$x_1'(t) = -x_1(t) + x_2(t), \quad x_2'(t) = x_1(t) - x_2(t).$$

Find the solutions $\mathbf{x}(t)$ to this differential equation.

(3) Let V be the subset of all $\mathbf{x}(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \end{bmatrix}$ in $\mathbb{D}^2$ for which:

$$x_1'(t) = 2x_1(t) + x_2(t), \quad x_2'(t) = x_1(t) + 2x_2(t).$$

Find the solutions $\mathbf{x}(t)$ to this differential equation.

Answers:

(1) $\mathbf{x}(t) = -e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + e^{-t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$

(2) $\mathbf{x}(t) = C_1 \begin{bmatrix} 1 \\ 1 \end{bmatrix} + C_2 e^{-2t} \begin{bmatrix} 1 \\ -1 \end{bmatrix}$ for some constants C_1, C_2 in $\mathbb{R}$

(3) $\mathbf{x}(t) = C_1 e^t \begin{bmatrix} 1 \\ -1 \end{bmatrix} + C_2 e^{3t} \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ for some constants C_1, C_2 in $\mathbb{R}$