Tutorial Worksheet #8
Friday, March 13

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§3.1 #3) Using a cofactor expansion across the first row, compute the determinant of the matrix
$$\begin{bmatrix} 2 & -2 & 3 \\ 3 & 1 & 2 \\ 1 & 3 & -1 \end{bmatrix}.$$

2. (§3.1 #9) Compute the following determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation:
$$\begin{vmatrix} 4 & 0 & 0 & 5 \\ 1 & 7 & 2 & -5 \\ 3 & 0 & 0 & 0 \\ 8 & 3 & 1 & 7 \end{vmatrix}.$$

3. (§3.1 #10) Compute the following determinant by cofactor expansion. At each step, choose a row or column that involves the least amount of computation:
$$\begin{vmatrix} 1 & -2 & 5 & 2 \\ 0 & 0 & 3 & 0 \\ 2 & -4 & -3 & 5 \\ 2 & 0 & 3 & 5 \end{vmatrix}.$$

4. (§3.1 #33) Verify that $\det EA = (\det E)(\det A)$, where E is the elementary matrix $E = \begin{bmatrix} 1 & k \\ 0 & 1 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

5. (§3.1 #35) Verify that $\det EA = (\det E)(\det A)$, where E is the elementary matrix $E = \begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$ and $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$.

6. (§3.1 #38) Let $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$ and let k be a scalar. Find a formula that relates $\det(kA)$ to k and $\det(A)$.

7. (§3.2 #5) Find the determinant
$$\begin{vmatrix} 1 & 5 & -4 \\ -1 & -4 & 5 \\ -2 & -8 & 7 \end{vmatrix}$$
by row reduction to echelon form.

8. (§3.2 #15) Suppose $\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$. Find

$$\begin{vmatrix} a & b & c \\ d & e & f \\ 3g & 3h & 3i \end{vmatrix}.$$

9. (§3.2 #21) Use the determinant to determine if the matrix

$$\begin{bmatrix} 2 & 6 & 0 \\ 1 & 3 & 2 \\ 3 & 9 & 2 \end{bmatrix}$$

is invertible.

10. (§3.2 #23) Use the determinant to determine if the matrix

$$\begin{bmatrix} 2 & 0 & 0 & 6 \\ 1 & -7 & -5 & 0 \\ 3 & 8 & 6 & 0 \\ 0 & 7 & 5 & 4 \end{bmatrix}$$

is invertible.

11. (§3.2 #25) Use the determinant to determine if the set of vectors

$$\begin{bmatrix} 7 \\ -4 \\ -6 \end{bmatrix}, \begin{bmatrix} -8 \\ 5 \\ 7 \end{bmatrix}, \begin{bmatrix} 7 \\ 0 \\ -5 \end{bmatrix}$$

is linearly independent.

12. (§3.2 #32) Suppose that A is a square matrix such that $\det A^3 = 0$. Explain why A cannot be invertible.

13. (§3.2 #34) Let A and P be square matrices, with P invertible. Show that

$$\det(PAP^{-1}) = \det A.$$

14. (§3.3 #5) Use Cramer's Rule to compute the solution of the system

$$\begin{aligned} x_1 + x_2 &= 3 \\ -3x_1 + 2x_3 &= 0 \\ x_2 - 2x_3 &= 2. \end{aligned}$$

15. (§3.3 #11) Compute the adjugate (classical adjoint) of the matrix

$$\begin{bmatrix} 0 & -2 & -1 \\ 5 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix},$$

and then use this to give the inverse of the matrix.

16. (§3.3 #21) Find the area of the parallelogram whose vertices are $(-2, 0), (0, 3), (1, 3), (-1, 0)$.