Tutorial Worksheet #7
Friday, March 6

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§2.8 #13) Find a non-zero vector in Nul A and a non-zero vector in Col A for
$$A = \begin{bmatrix} 3 & 2 & 1 & -5 \\ -9 & -4 & 1 & 7 \\ 9 & 2 & -5 & 1 \end{bmatrix}.$$

2. (§2.8 #17) Is the set
$$\left\{ \begin{bmatrix} 0 \\ 1 \\ -2 \end{bmatrix}, \begin{bmatrix} 5 \\ -7 \\ 4 \end{bmatrix}, \begin{bmatrix} 6 \\ 3 \\ 5 \end{bmatrix} \right\}$$
a basis for $\mathbb{R}^3$? Justify your answer.

3. (§2.8 #24) Below is a matrix A and an echelon form for A:
$$A = \begin{bmatrix} -3 & 9 & -2 & -7 \\ 2 & -6 & 4 & 8 \\ 3 & -9 & -2 & 2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & -3 & 6 & 9 \\ 0 & 0 & 4 & 5 \\ 0 & 0 & 0 & 0 \end{bmatrix}.$$
Find a basis for Col A and a basis for Nul A.

4. (§2.8 #25) Below is a matrix A and an echelon form for A:
$$A = \begin{bmatrix} 1 & 4 & 8 & -3 & -7 \\ -1 & 2 & 7 & 3 & 4 \\ -2 & 2 & 9 & -5 & -2 \\ 3 & 6 & 9 & -5 & -2 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 4 & 8 & 0 & 5 \\ 0 & 2 & 5 & 0 & -1 \\ 0 & 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$
Find a basis for Col A and a basis for Nul A.

5. (§2.8 #27) Construct a non-zero 3×3 matrix A and a non-zero vector $\mathbf{b}$ such that $\mathbf{b}$ is in Col A, but $\mathbf{b}$ is not the same as any one of the columns of A.

6. (§2.8 #29) Construct a non-zero 3×3 matrix A and a non-zero vector $\mathbf{b}$ such that $\mathbf{b}$ is in Nul A.

7. (§2.8 #31) Suppose F is a 5×5 matrix whose column space is not equal to $\mathbb{R}^5$. What can you say about Nul F? Justify your answer.

8. (§2.8 #35) What can you say about Nul B when B is a 5×4 matrix with linearly independent columns? Justify your answer.

9. (§2.9 #5) The vector $\mathbf{x} = \begin{bmatrix} 4 \\ 10 \\ -7 \end{bmatrix}$ is in a subspace H with a basis $\mathcal{B} = \{\mathbf{b_1}, \mathbf{b_2}\}$, where

$$\mathbf{b_1} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix} \quad \text{and} \quad \mathbf{b_2} = \begin{bmatrix} -3 \\ -7 \\ 5 \end{bmatrix}.$$

Find the $\mathcal{B}$-coordinate vector of $\mathbf{x}$.

10. (§2.9 #11) Below is a matrix A and an echelon form for A:

$$A = \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 2 & 5 & -8 & 4 & 3 \\ -3 & -9 & 9 & -7 & -2 \\ 3 & 10 & -7 & 11 & 7 \end{bmatrix} \rightsquigarrow \begin{bmatrix} 1 & 2 & -5 & 0 & -1 \\ 0 & 1 & 2 & 4 & 5 \\ 0 & 0 & 0 & 1 & 2 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}.$$

Find a basis for Col A and a basis for Nul A, and then state the dimensions of these subspaces.

11. (§2.9 #14) Find a basis for the subspace spanned by the following vectors:

$$\begin{bmatrix} 1 \\ -1 \\ -2 \\ 5 \end{bmatrix}, \begin{bmatrix} 2 \\ -3 \\ -1 \\ 6 \end{bmatrix}, \begin{bmatrix} 0 \\ 2 \\ -6 \\ 8 \end{bmatrix}, \begin{bmatrix} -1 \\ 4 \\ -7 \\ 7 \end{bmatrix}, \begin{bmatrix} 3 \\ -8 \\ 9 \\ -5 \end{bmatrix}.$$

What is the dimension of the subspace?

12. (§2.9 #15) Suppose a 3×5 matrix A has three pivot columns. Is Col $A = \mathbb{R}^3$? Is Nul $A = \mathbb{R}^2$? Explain your answers.

13. (§2.9 #21) If the rank of a 7×6 matrix A is 4, what is the dimension of the solution space $A\mathbf{x} = \mathbf{0}$? Justify your answer.

14. (§2.9 #25) Let A be an $n \times p$ matrix whose column space is p-dimensional. Explain why the columns of A must be linearly independent.