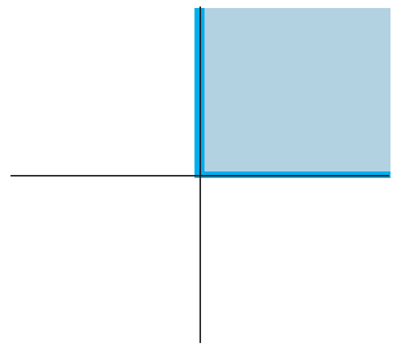
Tutorial Worksheet #6
Friday, February 28

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

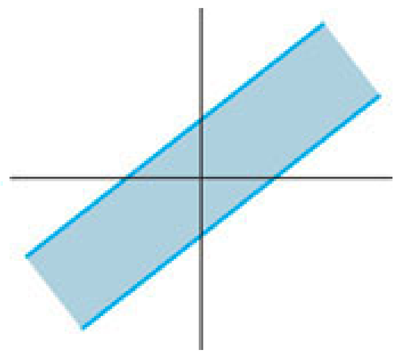
Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald. Denote the identity matrix by I. Also, for exercises from Section 2.3, assume all matrices are $n \times n$ unless otherwise specified.

1. (§2.3 #1) Determine if $\begin{bmatrix} 5 & 7 \\ -3 & -6 \end{bmatrix}$ is invertible. Use as few calculations as possible. Justify your answer.
2. (§2.3 #5) Determine if $\begin{bmatrix} 0 & 3 & -5 \\ 1 & 0 & 2 \\ -4 & -9 & 7 \end{bmatrix}$ is invertible. Use as few calculations as possible. Justify your answer.
3. (§2.3 #17) If A is invertible, then the columns of A^{-1} are linearly independent. Explain why.
4. (§2.3 #19) If the columns of a 7×7 matrix D are linearly independent, what can you say about solutions of $D\mathbf{x} = \mathbf{b}$? Why?
5. (§2.3 #21) If the equation $G\mathbf{x} = \mathbf{y}$ has more than one solution for some $\mathbf{y}$ in $\mathbb{R}^n$, can the columns of G span $\mathbb{R}^n$? Why or why not?
6. (§2.3 #26) Explain why the columns of A^2 span $\mathbb{R}^n$ whenever the columns of A are linearly independent.
7. (§2.3 #28) Show that if AB is invertible, so is B.
8. (§2.3 #29) If A is an $n \times n$ matrix and the equation $A\mathbf{x} = \mathbf{b}$ has more than one solution for some $\mathbf{b}$ in $\mathbb{R}^n$, then the transformation $\mathbf{x} \mapsto A\mathbf{x}$ is not one-to-one. What else can you say about this transformation? Justify your answer.
9. (§2.3 #33) The transformation $T : \mathbb{R}^2 \to \mathbb{R}^2$ defined by $T(x_1, x_2) = (-5x_1 + 9x_2, 4x_1 - 7x_2)$ is linear. Show that T is invertible and find a formula for T^{-1}.
10. (§2.3 #37) Suppose T and U are linear transformations from $\mathbb{R}^n$ to $\mathbb{R}^n$ such that $T(U\mathbf{x}) = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$. Is it true that $U(T\mathbf{x}) = \mathbf{x}$ for all $\mathbf{x}$ in $\mathbb{R}^n$. Why or why not?

11. (§2.8 #1) The figure below displays a set H in $\mathbb{R}^2$. Assume the set includes the bounding lines. Give a specific reason why the set H is *not* a subspace of $\mathbb{R}^2$.



12. (§2.8 #3) The figure below displays a set H in $\mathbb{R}^2$. Assume the set includes the bounding lines. Give a specific reason why the set H is *not* a subspace of $\mathbb{R}^2$.



13. (§2.8 #5) Let $\mathbf{v_1} = \begin{bmatrix} 2 \\ 3 \\ -5 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -4 \\ -5 \\ 8 \end{bmatrix}$, and $\mathbf{w} = \begin{bmatrix} 8 \\ 2 \\ -9 \end{bmatrix}$. Determine if $\mathbf{w}$ is in the subspace of $\mathbb{R}^3$ generated by $\mathbf{v_1}$ and $\mathbf{v_2}$.

14. (§2.8 #7) Let $\mathbf{v_1} = \begin{bmatrix} 2 \\ -8 \\ 6 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -3 \\ 8 \\ -7 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} -4 \\ 6 \\ -7 \end{bmatrix}, \mathbf{p} = \begin{bmatrix} 6 \\ -10 \\ 11 \end{bmatrix}$, and $A = \begin{bmatrix} \mathbf{v_1} & \mathbf{v_2} & \mathbf{v_3} \end{bmatrix}$.

    (a) How many vectors are in $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$?

    (b) How many vectors are in Col A?

    (c) Is $\mathbf{p}$ in Col A? Why or why not?

15. (§2.8 #9) With A and $\mathbf{p}$ as in Exercise 14, determine if $\mathbf{p}$ is in Nul A.