Tutorial Worksheet #5

Friday, February 14

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald. Denote the identity matrix by I.

1. Let $T : \mathbb{R}^p \to \mathbb{R}^n$ and $S : \mathbb{R}^n \to \mathbb{R}^m$ be linear transformations. The composition of S and T is the transformation $\mathbb{R}^p \to \mathbb{R}^n$ defined by
$$(S \circ T)(\mathbf{x}) = S(T(\mathbf{x})).$$
Show that $S \circ T$ is linear using the definition of a linear transformation.

2. (§2.1 #5) Compute the product AB in two ways: (a) by the definition, where $A\mathbf{b_1}$ and $A\mathbf{b_2}$ are computed separately, and (b) by the row-column rule for computing AB:
$$A = \begin{bmatrix} -1 & 2 \\ 5 & 4 \\ 2 & -3 \end{bmatrix} \quad \text{and} B = \begin{bmatrix} 3 & -2 \\ -2 & 1 \end{bmatrix}.$$

3. (§2.1 #9) Let $A = \begin{bmatrix} 2 & 5 \\ -3 & 1 \end{bmatrix}$ and $B = \begin{bmatrix} 4 & -5 \\ 3 & k \end{bmatrix}$. What value(s) of k, if any, will make $AB = BA$?

4. (§2.1 #17) If $A = \begin{bmatrix} 1 & -2 \\ -2 & 5 \end{bmatrix}$ and $AB = \begin{bmatrix} -1 & 2 & -1 \\ 6 & -9 & 3 \end{bmatrix}$, determine the first and second columns of B.

5. (§2.1 #19) Suppose the third column of B is the sum of the first two columns. What can you say about the third column of AB? Why?

6. (§2.1 #21) Suppose the last column of AB is entirely zero but B itself has no column of zeros. What can you say about the columns of A?

7. (§2.1 #22) Show that if the columns of B are linearly dependent, then so are the columns of AB.

8. (§2.1 #34) Give a formula for $(AB\mathbf{x})^T$, where $\mathbf{x}$ is a vector and A and B are matrices of appropriate sizes.

9. (§2.2 #1) Find the inverse of the matrix $\begin{bmatrix} 8 & 6 \\ 5 & 4 \end{bmatrix}$.

10. (§2.2 #5) Use the inverse found in the previous exercise to solve the system
$$\begin{aligned} 8x_1 + 6x_2 &= 2 \\ 5x_1 + 4x_2 &= -1. \end{aligned}$$

11. (§2.2 #8) Use matrix algebra to show that if A is invertible and D satisfies $AD = I$, then $D = A^{-1}$.

12. (§2.2 #13) Suppose $AB = AC$, where B and C are $n \times p$ matrices and A is invertible. Show that $B = C$. Is this true, in general, when A is not invertible.

13. (§2.2 #15) Suppose A, B, and C are invertible $n \times n$ matrices. Show that ABC is also invertible by producing a matrix D such that $(ABC)D = I$ and $D(ABC) = I$.

14. (§2.2 #16) Suppose A and B are $n \times n$ matrices, B is invertible, and AB is invertible. Show that A is invertible. [*Hint:* Let $C = AB$, and solve this equation for A.]

15. (§2.2 #31) Find the inverse of $\begin{bmatrix} 1 & 0 & -2 \\ -3 & 1 & 4 \\ 2 & -3 & 4 \end{bmatrix}$, if it exists. Use the algorithm introduced in Section 2.2.

16. (§2.2 #35) Let $A = \begin{bmatrix} -2 & -7 & -9 \\ 2 & 5 & 6 \\ 1 & 3 & 4 \end{bmatrix}$. Find the third column of A^{-1} without computing the other columns.