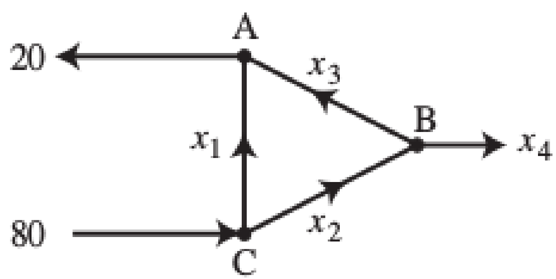
Tutorial Worksheet #3
Friday, January 31

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§1.6 #1) Suppose an economy has only two sectors, Goods and Services. Each year, Goods sells 80% of its output to Services and keeps the rest, while Services sells 70% of its output to Goods and retains the rest. Find equilibrium prices for the annual outputs of the Goods and Services sectors that make each sector's income match its expenditures.

2. (§1.6 #11) Consider the network shown below:



   (a) Find the general flow pattern of the network.

   (b) Assuming that the flows are all non-negative, what is the largest possible value for x_3?

3. (§1.7 #1) Determine if the vectors $\mathbf{u} = \begin{bmatrix} 5 \\ 0 \\ 0 \end{bmatrix}, \mathbf{v} = \begin{bmatrix} 7 \\ 2 \\ -6 \end{bmatrix}, \mathbf{w} = \begin{bmatrix} 9 \\ 4 \\ -8 \end{bmatrix}$ are linearly independent. Justify your answer.

4. (§1.7 #7) Determine if the columns of the matrix

$$\begin{bmatrix} 1 & 4 & -3 & 0 \\ -2 & -7 & 5 & 1 \\ -4 & -5 & 7 & 5 \end{bmatrix}$$

   form a linearly independent set. Justify your answer.

5. (§1.7 #9) Let $\mathbf{v_1} = \begin{bmatrix} 1 \\ -3 \\ 2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -3 \\ 9 \\ -6 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 5 \\ -7 \\ h \end{bmatrix}$.

   (a) For what value(s) of h is $\mathbf{v_3}$ in Span$\{\mathbf{v_1}, \mathbf{v_2}\}$? Justify.

   (b) For what value(s) of h is $\{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ linearly dependent? Justify.

6. (§1.7 #13) Find the value(s) of h for which the vectors

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ 5 \\ -3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -2 \\ -9 \\ 6 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 3 \\ h \\ -9 \end{bmatrix}$$

are linearly dependent.

7. (§1.6 #14) Find the value(s) of h for which the vectors

$$\mathbf{v_1} = \begin{bmatrix} 1 \\ -1 \\ 3 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} -5 \\ 7 \\ 8 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 1 \\ 1 \\ h \end{bmatrix}$$

are linearly dependent.

8. (§1.7 #25) Describe the possible echelon forms of the 4×2 matrix $A = [\mathbf{a_1} \quad \mathbf{a_2}]$ such that $\mathbf{a_2}$ is not a multiple of $\mathbf{a_1}$.

9. (§1.7 #27) How many pivot columns must a 7×5 matrix have if its columns are linearly independent? Why?

10. (§1.7 #30)

    (a) Fill in the blank in the following statement: "If A is an $m \times n$ matrix, then the columns of A are linearly independent if and only if A has ??? pivot columns."

    (b) Explain why the statement in (a) is true.

11. (§1.7 #31) Given

$$A = \begin{bmatrix} 2 & 3 & 5 \\ -5 & 1 & -4 \\ -3 & -1 & -4 \\ 1 & 0 & 1 \end{bmatrix},$$

observe that the third column is the sum of the first two. Find a non-trivial solution of $A\mathbf{x} = \mathbf{0}$.