Tutorial Worksheet #2
Friday, January 24

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§1.3 #11) Determine if $\mathbf{b} = \begin{bmatrix} 2 \\ -1 \\ 6 \end{bmatrix}$ is a linear combination of $\mathbf{a_1} = \begin{bmatrix} 1 \\ -2 \\ 0 \end{bmatrix}$, $\mathbf{a_2} = \begin{bmatrix} 0 \\ 1 \\ 2 \end{bmatrix}$ and $\mathbf{a_3} = \begin{bmatrix} 5 \\ -6 \\ 8 \end{bmatrix}$.

2. (§1.3 #17) Let $\mathbf{a_1} = \begin{bmatrix} 1 \\ 4 \\ -2 \end{bmatrix}$, $\mathbf{a_2} = \begin{bmatrix} -2 \\ -3 \\ 7 \end{bmatrix}$, $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ h \end{bmatrix}$. For what value(s) of h is $\mathbf{b}$ in the plane spanned by $\mathbf{a_1}$ and $\mathbf{a_2}$?

3. (§1.3 #19) Give a geometric description of Span$\{\mathbf{v_1}, \mathbf{v_2}\}$ for the vectors

$$\mathbf{v_1} = \begin{bmatrix} 8 \\ 2 \\ -6 \end{bmatrix} \quad \text{and} \quad \mathbf{v_2} = \begin{bmatrix} 12 \\ 3 \\ -9 \end{bmatrix}.$$

4. (§1.3 #25) Let $A = \begin{bmatrix} 1 & 0 & -4 \\ 0 & 3 & -2 \\ -2 & 6 & 3 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} 4 \\ 1 \\ -4 \end{bmatrix}$. Denote the columns of A by $\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}$, and let $W = \text{Span}\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$.
   (a) Is $\mathbf{b}$ in $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$? How many vectors are in $\{\mathbf{a_1}, \mathbf{a_2}, \mathbf{a_3}\}$?
   (b) Is $\mathbf{b}$ in W? How many vectors are in W?
   (c) Show that $\mathbf{a_1}$ is in W.

5. (§1.3 #34 a) Use the vector $\mathbf{u} = (u_1, \ldots, u_n)$ to verify that

$$\mathbf{u} + (-\mathbf{u}) = \mathbf{0}.$$

6. (§1.4 #12) Given

$$A = \begin{bmatrix} 1 & 2 & 1 \\ -3 & -1 & 2 \\ 0 & 5 & 3 \end{bmatrix}, \mathbf{b} = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix},$$

write the augmented matrix for the linear system that corresponds to the matrix equation $A\mathbf{x} = \mathbf{b}$. Then solve the system and write the solution as a vector.

7. (§1.4 #14) Let $\mathbf{u} = \begin{bmatrix} 2 \\ -3 \\ 2 \end{bmatrix}$ and $A = \begin{bmatrix} 5 & 8 & 7 \\ 0 & 1 & -1 \\ 1 & 3 & 0 \end{bmatrix}$. Is $\mathbf{u}$ in the subset of $\mathbb{R}^3$ spanned by the columns of A? Why or why not?

8. (§1.4 #16) Let $A = \begin{bmatrix} 1 & -3 & -4 \\ -3 & 2 & 6 \\ 5 & -1 & -8 \end{bmatrix}$ and $\mathbf{b} = \begin{bmatrix} b_1 \\ b_2 \\ b_3 \end{bmatrix}$. Show that the equation $A\mathbf{x} = \mathbf{b}$ does not have a solution for all possible $\mathbf{b}$, and describe the set of all $\mathbf{b}$ for which $A\mathbf{x} = \mathbf{b}$ *does* have a solution.

9. (§1.4 #17 & 19) Let
$$A = \begin{bmatrix} 1 & 3 & 0 & 3 \\ -1 & -1 & -1 & 1 \\ 0 & -4 & 2 & -8 \\ 2 & 0 & 3 & -1 \end{bmatrix}.$$
   (a) How many rows of A contain a pivot position? Does the equation $A\mathbf{x} = \mathbf{b}$ have a solution for each $\mathbf{b}$ in $\mathbb{R}^4$?
   (b) Do the columns of A span $\mathbb{R}^4$?
   (c) Can each vector in $\mathbb{R}^4$ be written as a linear combination of the columns of the matrix A?

10. (§1.4 #32) Could a set of three vectors in $\mathbb{R}^4$ span all of $\mathbb{R}^4$? Explain. What about n vectors in $\mathbb{R}^m$ when n is less than m?

11. (§1.4 #35) Let A be a 3×4 matrix, let $\mathbf{y_1}$ and $\mathbf{y_2}$ be vectors in $\mathbb{R}^3$, and let $\mathbf{w} = \mathbf{y_1} + \mathbf{y_2}$. Suppose $\mathbf{y_1} = A\mathbf{x_1}$ and $\mathbf{y_2} = A\mathbf{x_2}$ for some vectors $\mathbf{x_1}$ and $\mathbf{x_2}$ in $\mathbb{R}^4$. What fact allows you to conclude that the system $A\mathbf{x} = \mathbf{w}$ is consistent?

12. (#1.5 #7) Describe all solutions of $A\mathbf{x} = \mathbf{0}$ in parametric vector form where A is row equivalent to the matrix
$$\begin{bmatrix} 1 & 3 & -3 & 7 \\ 0 & 1 & -4 & 5 \end{bmatrix}.$$

13. (§1.5 #13) Suppose the solution set of a certain system of linear equations can be described as $x_1 = 5 + 4x_3, x_2 = -2 - 7x_3$, with x_3 free. Use vectors to describe this set as a line in $\mathbb{R}^3$.

14. (§1.5 #17) Describe and compare the solution sets of $x_1 + 9x_2 - 4x_3 = 0$ and $x_1 + 9x_2 - 4x_3 = -2$.

15. (§1.5 #19) Find the parametric equation of the line through $\mathbf{a} = \begin{bmatrix} -2 \\ 0 \end{bmatrix}$ parallel to $\mathbf{b} = \begin{bmatrix} -5 \\ 3 \end{bmatrix}$.

16. (§1.5 #25) Let $\mathbf{w}$ be any solution of $A\mathbf{x} = \mathbf{b}$, and define $\mathbf{v}_h = \mathbf{w} - \mathbf{p}$ where $\mathbf{p}$ is a solution to $A\mathbf{x} = \mathbf{b}$. Show that $\mathbf{v}_h$ is a solution of $A\mathbf{x} = \mathbf{0}$.

17. (§1.5 #40) Let A be an $m \times n$ matrix, and let $\mathbf{u}$ and $\mathbf{v}$ be vectors in $\mathbb{R}^n$ with the property that $A\mathbf{u} = \mathbf{0}$ and $A\mathbf{v} = \mathbf{0}$. Explain why $A(\mathbf{u} + \mathbf{v})$ must be the zero vector. Then explain why $A(c\mathbf{u} + d\mathbf{v}) = \mathbf{0}$ for each pair of scalars c and d.