# Tutorial Worksheet #10
## Friday, March 27

**Instructions:** Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

**Exercises:** From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§5.3 #2) Let $A = PDP^{-1}$ where
$$P = \begin{bmatrix} 2 & -3 \\ -3 & 5 \end{bmatrix} \quad \text{and} \quad D = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}.$$
Compute $A^4$.

2. (§5.3 #5) The following matrix $A$ is factored in the form $PDP^{-1}$. Use the Diagonalization Theorem to find the eigenvalues of $A$ and a basis for each eigenspace.
$$A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix} = \begin{bmatrix} 1 & 1 & 2 \\ 1 & 0 & -1 \\ 1 & -1 & 0 \end{bmatrix} \begin{bmatrix} 5 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1/4 & 1/2 & 1/4 \\ 1/4 & 1/2 & -3/4 \\ 1/4 & -1/2 & 1/4 \end{bmatrix}$$

3. (§5.3 #8) Diagonalize the matrix
$$A = \begin{bmatrix} 5 & 1 \\ 0 & 5 \end{bmatrix},$$
if possible.

4. (§5.3 #10) Diagonalize the matrix
$$A = \begin{bmatrix} 2 & 3 \\ 4 & 1 \end{bmatrix},$$
if possible.

5. (§5.3 #13) Diagonalize the matrix
$$A = \begin{bmatrix} 2 & 2 & -1 \\ 1 & 3 & -1 \\ -1 & -2 & 2 \end{bmatrix},$$
if possible. The eigenvalues of $A$ are $\lambda = 5, 1$.

6. (§5.3 #17) Diagonalize the matrix
$$A = \begin{bmatrix} 4 & 0 & 0 \\ 1 & 4 & 0 \\ 0 & 0 & 5 \end{bmatrix},$$
if possible.

7. (§5.3 #23) $A$ is a $5 \times 5$ matrix with two eigenvalues. One eigenspace is three-dimensional, and the other eigenspace is two-dimensional. Is $A$ diagonalizable? Why?

8. (§5.3 #25) $A$ is a $4 \times 4$ matrix with three eigenvalues. One eigenspace is one-dimensional, and one of the other eigenspaces is two-dimensional. Is it possible that $A$ is *not* diagonalizable? Justify your answer.

9. (§5.3 #27) Show that if $A$ is both diagonalizable and invertible, then so is $A^{-1}$.

10. (§5.3 #28) Show that if an $n \times n$ matrix $A$ has $n$ linearly independent eigenvectors, then so does $A^T$. [*Hint:* Use the Diagonalization Theorem.]