Tutorial Worksheet #1

Friday, January 17

Instructions: Work through the following exercises. *Fully show and explain all of your work.* Remember to use good notation and full sentences.

Exercises: From the textbook *Linear Algebra And Its Applications*, fifth edition, by David C. Lay, Steven R. Lay, Judi J. McDonald.

1. (§1.1 #11 & 13) Solve the following linear systems.

   (a)

$$\begin{aligned} x_2 + 4x_3 &= -5 \\ x_1 + 3x_2 + 5x_3 &= -2 \\ 3x_1 + 7x_2 + 7x_3 &= 6 \end{aligned}$$

   (b)

$$\begin{aligned} x_1 - 3x_3 &= 8 \\ 2x_1 + 2x_2 + 9x_3 &= 7 \\ x_2 + 5x_3 &= -2 \end{aligned}$$

2. (§1.1 #16) Determine if the following system is consistent:

$$\begin{aligned} x_1 - 2x_4 &= -3 \\ 2x_2 + 2x_3 &= 0 \\ x_3 + 3x_4 &= 1 \\ -2x_1 + 3x_2 + 2x_3 + x_4 &= 5 \end{aligned}$$

3. (§1.1 #19) Determine the value(s) of h such that the matrix

$$\begin{bmatrix} 1 & h & 4 \\ 3 & 6 & 8 \end{bmatrix}$$

   is the augmented matrix of a consistent linear system.

4. (§1.1 #27) Suppose the system below is consistent for all possible values of f and g. What can you say about the coefficients c and d? Justify your answer.

$$\begin{aligned} x_1 + 3x_2 &= f \\ cx_1 + dx_2 &= g \end{aligned}$$

5. (§1.2 #1) Determine which of the following matrices are in reduced echelon form and which others are only in echelon form:

   (a) $A = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \end{bmatrix}$

(b) $B = \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(c) $C = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$

(d) $D = \begin{bmatrix} 1 & 1 & 0 & 1 & 1 \\ 0 & 2 & 0 & 2 & 2 \\ 0 & 0 & 0 & 3 & 3 \\ 0 & 0 & 0 & 0 & 4 \end{bmatrix}$

6. (§1.2 #8 & 12) Find the general solutions of the systems whose augmented matrix is given:

   (a)
   $$\begin{bmatrix} 1 & 4 & 0 & 7 \\ 2 & 7 & 0 & 10 \end{bmatrix}$$

   (b)
   $$\begin{bmatrix} 1 & -7 & 0 & 6 & 5 \\ 0 & 0 & 1 & -2 & -3 \\ -1 & 7 & -4 & 2 & 7 \end{bmatrix}$$

7. (§1.2 #19) Choose h and k such that the following system has (a) no solution, (b) a unique solution, and (c) many solutions. Give separate answers for each part.
   $$\begin{aligned} x_1 + hx_2 &= 2 \\ 4x_1 + 8x_2 &= k \end{aligned}$$

8. (§1.2 #24) Suppose a system of linear equations has a 3×5 augmented matrix whose fifth column is a pivot column. Is the system consistent? Why (or why not)?

9. (§1.2 #25) Suppose the coefficient matrix of a system of linear equations has a pivot position in every row. Explain why the system is consistent.

10. (§1.2 #31) A system of linear equations with more equations than unknowns is sometimes called an *overdetermined system*. Can such a system be consistent? Illustrate your answer with a specific system of three equations in two unknowns.