# Appendix B: Complex Numbers

**Question:** What are the real solutions to $x^2 - 1$?

**Question:** What are the solutions to $x^2 + 1 = 0$?

**Note:** To handle $\sqrt{-1}$, we

**Definition:** A **complex number** is a number of the form

$$z = a + bi$$

where $a$ and $b$ are real numbers. We denote the set of all complex numbers by $\mathbb{C}$. Also,

**Notes:**

1. A real number is a complex number:

2. $\mathbb{C} \longleftrightarrow \mathbb{R}^2$

**Operations:** Let $z = a + bi$ and $w = c + di$ be complex numbers. Then

1. $z + w =$

2. $zw =$

3. The **conjugate** of $z$ is

4. The **absolute value** (or **modulus**) of $z$ is

5. If $z \neq 0$ (i.e., $z \neq 0 + 0i$), then the **multiplicative inverse** of $z$ is

**Examples:** Let $z = 1 + 3i$ and $w = 4 - 2i$.

1. $z + w$

2. $zw$

3. $\overline{z}$

4. $z\overline{z}$

5. $|z|$

6. $\frac{w}{z}$

**Useful Properties:** Let $z$ and $w$ be complex numbers.

1. $\overline{\overline{z}} = z$

2. $\overline{z+w} =$

3. $\overline{zw} =$

4. $z\overline{z} =$

5. $|zw| =$

6. $|z+w|$

7. $z = a + bi \implies$

**Polar Coordinates:** Given the complex number $z = a+bi$, let $\varphi$ be the angle between the positive real axis and $(a, b)$, where $-\pi < \varphi \leq \pi$.

**De Moivre's Theorem:** Let $z = a + bi = r(\cos\varphi + i\sin\varphi)$ where $r = |z|$. Then

**Example:** Find $z^7$ where $z = \sqrt{3} + i$.