Chapter 6: Orthogonality and Least Squares

§6.1 – Orthogonality (Continued)

Example: Let $\mathbf{u} = \begin{bmatrix} 2 \\ -1 \\ 3 \end{bmatrix}$ and $\mathbf{a} = \begin{bmatrix} 4 \\ -1 \\ 2 \end{bmatrix}$.

Applications of Orthogonality:

Pythagorean Theorem: If $\mathbf{u}$ and $\mathbf{v}$ are orthogonal vectors in $\mathbb{R}^n$, then

$$||\mathbf{u} + \mathbf{v}||^2 = ||\mathbf{u}||^2 + ||\mathbf{v}||^2.$$

Theorem:

1. In $\mathbb{R}^2$, the distance between the point $P_0(x_0, y_0)$ and the line $ax + by + c = 0$ is

2. In $\mathbb{R}^3$, the distance between the point $P_0(x_0, y_0, z_0)$ and the plane $ax + by + cz + d = 0$ is

Example: Find the distance between $(1, -4, -3)$ and the plane $2x - 3y + 6z = -1$.

Definition: Let W be a subspace of $\mathbb{R}^n$. The **orthogonal complement** of W is

Theorem: If W is a subspace of $\mathbb{R}^n$, then $W^\perp$ is also a subspace of $\mathbb{R}^n$.

Corollary: Let W be a subspace of $\mathbb{R}^n$. Then

Theorem: Let A be an $m \times n$ matrix.

Example: Let $A = \begin{bmatrix} 1 & -3 & 2 & -4 \\ -3 & 9 & -1 & 5 \\ 2 & -6 & 4 & -3 \\ -4 & 12 & 2 & 7 \end{bmatrix}$