Chapter 6: Orthogonality and Least Squares

§6.1 – Inner Product, Length, and Orthogonality (Continued)

Recall: If $\mathbf{u}, \mathbf{v}$ are non-zero vectors in $\mathbb{R}^2$ or $\mathbb{R}^3$, then

Definition: Two vectors $\mathbf{u}$ and $\mathbf{v}$ are in $\mathbb{R}^n$ are said to be **orthogonal** if $\mathbf{u} \cdot \mathbf{v} = 0$.

Examples:

1.

2. Let $\mathbf{u} = \begin{bmatrix} -2 \\ 3 \\ 1 \\ 4 \end{bmatrix}$ and $\mathbf{v} = \begin{bmatrix} 1 \\ 2 \\ 0 \\ -1 \end{bmatrix}$

Point-Normal Equations of Lines and Planes:

Example: $y = -6x + 11$ is an equation of a line in $\mathbb{R}^2$. By re-arranging, we have

$$6(x - 3) + (y + 7) = 0.$$

Similarly, $4(x-3)+2y-5(z-7)=0$ represents a plane in $\mathbb{R}^3$ through

Theorem:

1. If a, b are constants not both 0, then an equation
$$ax + by + c = 0$$
represents a line in $\mathbb{R}^2$ with normal $\mathbf{n} = (a, b)$.

2. If a, b, c are constants not both 0, then an equation
$$ax + by + cz + d = 0$$
represents a plane in $\mathbb{R}^3$ with normal $\mathbf{n} = (a, b, c)$.

Special Cases of the Theorem:

Orthogonal Projections:

Goal: To decompose a vector $\mathbf{u}$ into $\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$ where (i) $\mathbf{w_1} = c\mathbf{a}$ for a given vector $\mathbf{a}$, and (ii) $\mathbf{w_1}$ and $\mathbf{w_2}$ are orthogonal.

Projection Theorem: Let $\mathbf{u}$ and $\mathbf{a}$ be vectors in $\mathbb{R}^n$ with $\mathbf{a} \neq \mathbf{0}$. Then we can uniquely express

$$\mathbf{u} = \mathbf{w_1} + \mathbf{w_2}$$

where $\mathbf{w_1} = k\mathbf{a}$ and $\mathbf{w_2} \cdot \mathbf{a} = 0$.

Definition: Let $\mathbf{u}, \mathbf{a}$ be in $\mathbb{R}^n$ with $\mathbf{a} \neq \mathbf{0}$.

1. The **vector component** of $\mathbf{u}$ **along** $\mathbf{a}$ (or **orthogonal projection** of $\mathbf{u}$ on $\mathbf{a}$) is

2. The **vector component** of $\mathbf{u}$ **orthogonal** to $\mathbf{a}$ is