Chapter 5: Eigenvalues and Eigenvectors

§5.4 – Eigenvectors and Linear Transformations

Goal: To understand what diagonalization means for linear transformations.

Recall: An $n \times n$ matrix A is diagonalizable if and only if A is similar to a diagonal matrix. That is, A is diagonlizable if and only if $A = PDP^{-1}$ for some invertible matrix P and diagonal matrix D.

Examples:

1. Let $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$. Define $T : \mathbb{R}^3 \to \mathbb{R}^3$ by $T(\mathbf{x}) = A\mathbf{x}$.

   Notes:

   (a) The standard matrix of T is A. Thus, if

   $$\mathcal{E} = \left\{ \mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} \right\}$$

   is the standard basis for $\mathbb{R}^3$, we have

(b)

Now $\det(A - \lambda I) = -(\lambda - 2)^2(\lambda - 1)$ (worked out in a past class!). Hence,

- A has eigenvalues

- The eigenspace for $\lambda_2 = 2$ has basis

- The eigenspace for $\lambda_1 = 1$ has basis

- We have 3 linearly independent eigenvectors and so A is

Observe: $\mathcal{B} = \{\mathbf{v_1}, \mathbf{v_2}, \mathbf{v_3}\}$ is an eigenvector basis for $\mathbb{R}^3$.

We have:

2. Define $T : \mathbb{R}^2 \to \mathbb{R}^2$ by $T(\mathbf{x}) = A\mathbf{x}$ where $A = \begin{bmatrix} 5 & -3 \\ -7 & 1 \end{bmatrix}$. Find a basis $\mathcal{B}$ for $\mathbb{R}^2$ such that $[T]_{\mathcal{B}}$ is diagonal.