Chapter 5: Eigenvalues and Eigenvectors

§5.3 – Diagonalization

Motivating Example: Let $A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}$. What do powers A^k look like?

Note: Let $D = \begin{bmatrix} 1 & 0 \\ 0 & 1/2 \end{bmatrix}$. Then $A = PDP^{-1}$ where $P = \begin{bmatrix} 3 & 1 \\ 2 & -1 \end{bmatrix}$. Now:

Fact:

Example: If $A = \begin{bmatrix} 2 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 3 \end{bmatrix}$, then $A^3 = \begin{bmatrix} 8 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 27 \end{bmatrix}$.

Goal: For an $n \times n$ matrix A, we would like to "replace" A with a diagonal matrix.

Definition: A square matrix A is said to be **diagonizable** if A is similar to a diagonal matrix; that is,

Question: How would we find such matrices P and D?

The Diagonalization Theorem: Let A be an $n \times n$ matrix. We have: A is diagonalizable if and only if A has n linearly independent eigenvectors. In fact,

Note/Definition: An $n \times n$ matrix A is diagonalizable if and only if there is a basis for $\mathbb{R}^n$ consisting entirely of eigenvectors of A. We call such a basis an

Examples: Diagonalize the following matrices, if possible.

1. $A = \begin{bmatrix} 0 & -1 \\ 1 & 0 \end{bmatrix}$

2. $A = \begin{bmatrix} 3 & 1 & 1 \\ 0 & 2 & 1 \\ 0 & 0 & 1 \end{bmatrix}$