# Chapter 5: Eigenvalues and Eigenvectors

## §5.2 – The Characteristic Equation

**The Invertible Matrix Theorem (continued):** Let $A$ be an $n \times n$ matrix. Then $A$ is invertible if and only if:

**Question:** How do we find eigenvalues?

**Note:** Let $A$ be an $n \times n$ matrix. Then:

**Definition:** Let $A$ be an $n \times n$ matrix.

1. $\det(A - \lambda I)$ is a polynomial in $\lambda$ of degree $n$ called the
2. $\det(A - \lambda I) = 0$ is called the
3. The **(algebraic) multiplicity** of the eigenvalue $\lambda$ is its multiplicity as a root

**Examples:**

1. $A = \begin{bmatrix} -2 & 1 \\ 1 & -2 \end{bmatrix}$

2. $A = \begin{bmatrix} -1 & 3 & 2 \\ 0 & 0 & 6 \\ 0 & 0 & 8 \end{bmatrix}$

3. $A = \begin{bmatrix} 0 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \end{bmatrix}$

4. $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$

**Definition:** Let $A$ and $B$ be $n \times n$ matrices. We say that $A$ is **similar** to $B$ if there is an invertible matrix $P$ such that

**Theorem:** If $n \times n$ matrices $A$ and $B$ are similar, then they have the same characteristic polynomial, and hence the same eigenvalues (with the same multiplicities).

**Note:** Matrices with the same eigenvalues are not necessarily similar matrices.

**True/False?** Are the following statements true or false? Justify your answer.

1. If an $n \times n$ matrix $A$ has $k$ distinct eigenvalues, then $\text{rank}(A) \geq k$.

2. If $A$ is a $4 \times 4$ matrix such that $A^4 = 0$, then $\lambda = 0$ is the only eigenvalue of $A$.