Chapter 5: Eigenvalues and Eigenvectors

§5.1 – Eigenvectors and Eigenvalues

Example: Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be the linear transformation with standard matrix

$$A = \begin{bmatrix} 0.8 & 0.3 \\ 0.2 & 0.7 \end{bmatrix}.$$

What is $T\left(\begin{bmatrix} 1 \\ -1 \end{bmatrix}\right)$?

Definition: Let A be an $n \times n$ matrix.

1. An **eigenvector** of A is a vector

2. A scalar λ is an **eigenvalue** of A if there is a

Examples: Let $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$.

1. $\mathbf{u} = \begin{bmatrix} 1 \\ 2 \end{bmatrix}$ is

   $\mathbf{v} = \begin{bmatrix} 1 \\ 1 \end{bmatrix}$ is

2. Show that -1 is an eigenvalue of A and find the corresponding eigenvectors.

Note: λ is an eigenvalue of an $n \times n$ matrix A

Definition: Let A be an $n \times n$ matrix. The nullspace of $A - \lambda I$ is called the **eigenspace** of

Note: The eigenspace equals

Example: $A = \begin{bmatrix} 5 & -6 & -6 \\ -1 & 4 & 2 \\ 3 & -6 & -4 \end{bmatrix}$ has $\lambda = 2$ as an eigenvalue. Find a basis for the corresponding eigenspace.

Useful Facts:

1. The eigenvalues of a triangular matrix are the entries

2. An $n \times n$ matrix A has $\lambda = 0$ as an eigenvalue

Example: $A = \begin{bmatrix} 4 & 2 & -8 \\ 0 & -1 & 6 \\ 0 & 0 & 0 \end{bmatrix}$ has eigenvalues

Theorem: If $\mathbf{v_1}, \ldots, \mathbf{v_p}$ are eigenvectors corresponding to *distinct* eigenvalues $\lambda_1, \ldots, \lambda_p$ of an $n \times n$ matrix A, then $\{\mathbf{v_1}, \ldots, \mathbf{v_p}\}$ is linearly independent.