Chapter 3: Determinants

§3.3 – Cramer's Rule, Volume & Linear Transformations (Cont.)

Part I - Cramer's Rule (continued):

The Classical Adjoint and Inverses:

Let A be an $n \times n$ invertible matrix.

Goal: Find a formula for A^{-1}.

We know: The jth column of A^{-1} is the solution to

$$A\mathbf{x} = \mathbf{e_j}$$

Cramer's Rule:

Theorem: Let A be an $n \times n$ invertible matrix. We have

Example: Let $A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 2 \\ 5 & 0 & 2 \end{bmatrix}$. Last week we found $\det(A) = 30 \neq 0$ and so A^{-1} exists. We also found:

Part II - Determinants and Geometry:

Example: Consider a parallelogram in $\mathbb{R}^2$:

The area depends on the base and height. What if we had:

Fact: Let $\mathbf{a_1}$ and $\mathbf{a_2}$ be non-zero vectors in $\mathbb{R}^2$. Then for any scalar c, the area of the parallelogram determined by $\mathbf{a_1}$ and $\mathbf{a_2}$ equals the area of the parallelogram determined by

Theorem: If A is a 2×2 matrix, the area of the parallelogram determined by the columns of A is $|\det A|$. If A is a 3×3 matrix, the volume of the parallelpiped determined by the columns of A is $|\det A|$.

Example: Find the area of the parallelogram determined by the points $(-2,-2), (0,3), (4,-1), (6,4)$.

Theorem:

1. Let $T : \mathbb{R}^2 \to \mathbb{R}^2$ be a linear transformation with standard matrix A. Let S be a parallelogram in $\mathbb{R}^2$. Then

2.