Chapter 3: Determinants

Section 3.1 – Introduction to Determinants (Continued)

Notation/Definition: Let $A = [a_{ij}]$ be an $n \times n$ matrix with $n \geq 2$.

1. $A_{ij} =$

2. The (i, j)-**cofactor** of A is

3. The **determinant** of A is

Theorem: The determinant of an $n \times n$ matrix $A = [a_{ij}]$ can be computed by a cofactor expansion across any row or down any column. That is,

Examples: Find the determinant of the following matrices.

1. $A = \begin{bmatrix} 2 & 1 & 3 \\ 5 & 0 & 0 \\ 1 & 2 & -1 \end{bmatrix}$

2. $A = \begin{bmatrix} 1 & 2 & 1 & 3 \\ 4 & 3 & 0 & -1 \\ 2 & -1 & 1 & 0 \\ 3 & 5 & 2 & 4 \end{bmatrix}$

3. $A = \begin{bmatrix} 5 & -7 & 8 & 9 & -6 \\ 0 & 2 & -3 & 7 & 3 \\ 0 & 0 & 1 & 5 & 0 \\ 0 & 0 & 0 & 4 & -1 \\ 0 & 0 & 0 & 0 & -1 \end{bmatrix}$

Theorem: The determinant of a triangular matrix is

Section 3.2 – Properties of Determinants

Question: How does a determinant change when we apply an elementary row operation?

Theorem: Let A be a square matrix.

1. If a multiple of one row of A is added to another row to produce the matrix B, then
2. If two rows of A are interchanged to produce the matrix B, then
3. If one row of A is multiplied by k to produce the matrix B, then

Examples:

1. $A = \begin{bmatrix} 2 & -1 & -1 \\ 1 & 3 & 2 \\ 1 & 1 & 3 \end{bmatrix}$

2. Assume

$$\begin{vmatrix} a & b & c \\ d & e & f \\ g & h & i \end{vmatrix} = 7$$

(a) $\begin{vmatrix} a & b & c \\ 5d & 5e & 5f \\ g & h & i \end{vmatrix}$

(b) $\begin{vmatrix} a & b & c \\ 3d+a & 3e+b & 3f+c \\ g & h & i \end{vmatrix}$

(c) $\begin{vmatrix} a & b & c \\ d+4g & e+4h & f+4i \\ g & h & i \end{vmatrix}$