Chapter 2: Matrix Algebra

Section 2.9 – Dimension and Rank (Continued)

Part II - The Dimension of a Subspace (Continued)

Definition: The **rank** of a matrix A is the

Example: Let $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$. Find: (a) $\dim \operatorname{Nul} A$; (b) $\dim \operatorname{Col} A$; (c) rank A

Note: For a matrix A:

1. $\dim(\operatorname{Nul} A) =$

2. $\dim(\operatorname{Col} A) =$

The Rank Theorem: If a matrix A has n columns, then

$$\operatorname{rank} A + \dim(\operatorname{Nul} A) =$$

True/False? A 3×3 matrix B can have Col $B =$ Nul B.

Part III - Rank and the Invertible Matrix Theorem

The Invertible Matrix Theorem (continued!): Let A be an $n \times n$ matrix. The following statements are each equivalent to the statement that A is an invertible matrix.

(m) The columns of A form a

(n) Col A

(o) $\dim(\operatorname{Col} A)$

(p) $\operatorname{rank} A$

(q) Nul A

(r) $\dim(\operatorname{Nul} A)$

Proof of $(p) \implies (r) \implies (q)$:

Chapter 3: Determinants

Goal: Investigate an easy criterion to determine if a matrix is invertible.....a numerical test!

Section 3.1 – Introduction to Determinants

Definition/Recall: For a 1×1 matrix $A = [a_{11}]$, the **determinant** of A is defined to be

$$\det A = a_{11}.$$

For a 2×2 matrix $A = \begin{bmatrix} a & b \\ c & d \end{bmatrix}$, we have

We want to generalize this for $n \times n$ matrices!

A Magic Show: Let

$$A = \begin{bmatrix} 1 & 2 & 0 \\ -1 & 3 & 2 \\ 5 & 0 & 2 \end{bmatrix}.$$

Let

- A_{ij} be the 2×2 submatrix obtained by removing row i and column j of A;
- $C_{ij} = (-1)^{i+j} \det(A_{ij})$.