Chapter 2: Matrix Algebra

Section 2.8 – Subspaces of $\mathbb{R}^n$ (Continued)

Recall: A **subspace** of $\mathbb{R}^n$ is any set H in $\mathbb{R}^n$ that has three properties:

1. $\mathbf{0}$ is in H;
2. For each $\mathbf{x}$ and $\mathbf{y}$ in H,
3. For each $\mathbf{x}$ in H and each scalar c, the vector

Also, given any $m \times n$ matrix $A = [\mathbf{a_1} \quad \cdots \quad \mathbf{a_n}]$, we have two special subspaces:

1. Col A
2. Nul A

Question: How can we efficiently describe a subspace?

Example: Let $H = \mathbb{R}^3$ which is a subspace of $\mathbb{R}^3$. Let

$$\mathbf{e_1} = \begin{bmatrix} 1 \\ 0 \\ 0 \end{bmatrix}, \mathbf{e_2} = \begin{bmatrix} 0 \\ 1 \\ 0 \end{bmatrix}, \mathbf{e_3} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix}.$$

Definition: A **basis** for a subspace H of $\mathbb{R}^n$ is a

Examples:

1. $\{\mathbf{e_1}, \ldots, \mathbf{e_n}\}$

2. $H = \text{Span}\left\{ \mathbf{v_1} = \begin{bmatrix} 1 \\ 1 \\ -2 \end{bmatrix}, \mathbf{v_2} = \begin{bmatrix} 1 \\ 3 \\ 2 \end{bmatrix}, \mathbf{v_3} = \begin{bmatrix} 1 \\ -1 \\ -6 \end{bmatrix} \right\}$

3. $H = \left\{ \begin{bmatrix} s \\ t \\ 0 \end{bmatrix} : s, t \text{ are in } \mathbb{R} \right\}$

4. $\left\{ \mathbf{x} = \begin{bmatrix} -4 \\ 6 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 2 \\ -3 \end{bmatrix} \right\}$

5. $\left\{ \mathbf{x} = \begin{bmatrix} 3 \\ -8 \\ 1 \end{bmatrix}, \mathbf{y} = \begin{bmatrix} 6 \\ 1 \\ 0 \end{bmatrix} \right\}$

6. Let $A = \begin{bmatrix} -2 & -5 & 8 & 0 & -17 \\ 1 & 3 & -5 & 1 & 5 \\ 3 & 11 & -19 & 7 & 1 \\ 1 & 7 & -13 & 5 & -3 \end{bmatrix}$. Let's find a basis for: (a) Nul A; (b) Col A.

Theorem: The pivot columns of a matrix A form a basis for Col A.

Warning: If $A \rightsquigarrow B = \text{RREF}(A)$, then possibly