Chapter 2: Matrix Algebra

Section 2.3 – Characterizations of Invertible Matrices

Recall: Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation with standard matrix A. Then T is invertible if and only if A^{-1} exists.

Food For Thought: Let $T : \mathbb{R}^n \to \mathbb{R}^n$ be a linear transformation with standard matrix A.

1. T is 1-1 $\implies$ T is invertible and maps onto $\mathbb{R}^n$.

2. T maps $\mathbb{R}^n$ onto $\mathbb{R}^n$ $\implies$ T^{-1} exists, T^{-1} is 1-1, and T^{-1} maps onto $\mathbb{R}^n$.

3. T is invertible $\implies$ T is 1-1 and maps onto $\mathbb{R}^n$.

Section 2.6 – The Leontief Input–Output Model

Goal: To see an application of inverses!

Example: Consider the following economy with three sectors:

Question: What amounts are consumed by the manufacturing sector if it produces 100 units?

In general, if x_1, x_2, x_3 denote the planned outputs of manufacturing, agriculture, and services, respectively, then

- $x_1 c_1$ = the intermediate demands of manufacturing;
- $x_2 c_2$ = the intermediate demands of agriculture;
- $x_3 c_3$ = the intermediate demands of services.

The Leontif Input-Output Model:

Question: In our example, if the final demand is 50 units for manufacturing, 30 units for agriculture, and 20 units for services, then what is the production level $\mathbf{x}$?

Theorem: Let C be the consumption matrix for an economy and $\mathbf{d}$ be the final demand. If C and $\mathbf{d}$ have non-negative entries and if each column sum of C is less than 1, then $(I - C)^{-1}$ exists and the production vector

$$\mathbf{x} = (I - C)^{-1}\mathbf{d}$$

has non-negative entries and is the unique solution of $\mathbf{x} = C\mathbf{x} + \mathbf{d}$.